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Categories and Particulars: Prototype Effects in Estimating Spatial Location

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A model of category effects on reports from memory is presented. The model holds that stimuli are represented at 2 levels of detail: a fine-grain value and a category. When memory is inexact but people must report an exact value, they use estimation processes that combine the remembered stimulus value with category information. The proposed estimation processes include truncation at category boundaries and weighting with a central (prototypic) category value. These processes introduce bias in reporting even when memory is unbiased, but nevertheless may improve overall accuracy (by decreasing the variability of reports). Four experiments are presented in which people report the location of a dot in a circle. Subjects spontaneously impose horizontal and vertical boundaries that divide the circle into quadrants. They misplace dots toward a central (prototypic) location in each quadrant, as predicted by the model. The proposed model has broad implications; notably, it has the potential to explain biases of the sort described in psychophysics (contraction bias and the bias captured by Weber's law) as well as asymmetries in similarity judgments, without positing distorted representations of physical scales.

In this article we propose a model of category effects found in reports from episodic memory, that is, reports of the what, when, and where of particular experiences. For example, a person may try to remember the particular properties of an object (e.g., its size and color) or where an object was located. When memory is inexact, people's reports are reconstructions, influenced by schematic or category information (cf. Bartlett, 1932; Brewer & Nakamura, 1984). If information is simply forgotten, a default value may be reported (e.g., the usual color of that sort of object, or a location central to the area where the object could be). If information is remembered, but inexactly, reports may be blends, intermediate between an actual stimulus value and a category value (cf. Belli, 1988).

At present, precise models of category effects on reports of particular experiences are lacking. In proposing such a model here, we begin with stimulus domains based on continuous physical dimensions; object height, temporal or spatial location, and so on. The assumptions of the model are the familiar ones implicit in the previous examples: that memory is hierarchically organized and inexact, and that, in reporting, people may draw from information at two levels (a particular value and a category). The model is novel in that it posits that reports of particular stimulus values are based on estimation procedures that take account of prior (category) information. One of these estimation processes, truncation resulting from category boundaries, was described earlier in Huttenlocher, Hedges, and Prohaska (1988). A second estimation process, weighting with a prototype (a central value in the category), is the focus in the present article.

The proposed uses of category information in estimation introduce systematic biases in reporting even when memory, although inexact, is not itself biased. While introducing bias, these uses of category information nevertheless may be rational; that is, they may improve the overall accuracy of reports by decreasing their variability. This function of categories—the adjustment of inexactly represented stimulus values in a way that may potentially yield more accurate estimates—has not, thus far, been explored.

In the next section, we describe the general form of the proposed model. (The mathematical formulation is presented in the Appendix.) Then we apply the model to the estimation of spatial location. Four experiments are presented in which people reproduce the location of a dot in a circle: The observed patterns of bias are those predicted by the model. The use of category information, in our experiments, improves the overall accuracy of reporting. After explicating the model and producing the evidence for it in the case of the representation of spatial location, we consider the application of the model to more general issues. Notably, the model has implications for claims of systematic distortion in the mental representation of values along physically measurable dimensions, including spatial location, based on biases in reporting (e.g., asymmetries in distance judgments, the biases described in psychophysics). The model shows that, at least in some cases, such biases in reporting can be explained without positing biases in the representation of physical stimulus values. In addition, the model has implications for arguments concerning the representation of category information in memory. In particular, at least for the purpose of estimation, models that posit explicit representation of category information (boundaries and prototypes) may yield a more natural explanation than do models that posit the implicit representation of categories as sets of exemplars.

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Category Model

Representation

The model makes the following claims about memory. The first is that the mental representation of physical scales is unbiased (in the sense described later). The second is that stimuli are represented at two levels of detail: a particular (fine-grain) value and a category. If representation is exact, information at the two levels is perfectly coordinated (i.e., redundant). However, if representation is inexact, the two levels provide nonredundant information (which can be combined in making estimates, as described later).

Fine-grain coding. In the model, the representation of a fine-grain stimulus value, no matter how inexact, is treated as unbiased. That is, an inexact representation in memory can be thought of as an underlying distribution of values centered at the true value. In recollection, a *fine-grain value* is sampled from this distribution, along with a notion of *fine-grain inexactness* that reflects the dispersion of the distribution.[1] The inexactness of fine-grain values depends on the degree of imprecision of encoding and on the extent of loss of particular information from memory.

Categories. A category is a bounded region that covers a range of fine-grain stimulus values. For example, a piece of cloth might be orange (a category) and one of a range of particular shades (a fine-grain value); in our experiments, where a dot is shown in a circle, the dot might be in the upper left quadrant (a category) at one of a range of particular coordinates (a fine-grain value).

A *boundary value* specifies an endpoint of the range of values included in a category. A boundary imposed by a subject (i.e., it is not physically present) is to some extent inexact. For example, orange has inexact boundaries with yellow and red at the two ends; the quadrants of the circle have inexact boundaries at the horizontal and vertical axes reflecting uncertainty in locating these axes by eye. Hence, associated with a boundary value is a notion of *boundary inexactness.* Boundary values are used in two ways. First, they are used to encode the category of a stimulus; a stimulus that falls in the range of boundary inexactness sometimes may fail to be encoded as being in the category. Second, for stimuli encoded as being in a category, boundary values of that category may be used to adjust inexact fine-grain values in estimation (as described later).

A category includes a presumed pattern of values across the region it encompasses; those values may be thought to form a relatively normal distribution (i.e., with most instances in the middle), a uniform distribution (i.e., with instances spread out across the range), and so on. This presumed pattern is captured by a central value (e.g., the mean or median of observed instances), a *prototype value.* For example, the category orange has a prototypic central shade; a quadrant has a prototypic central location. Associated with the prototype value is a notion of *prototype inexactness* that reflects the dispersion of instances over the category. The prototype value may be used to adjust inexact fine-grain values in estimation (as described later).

Estimation

The model makes the following claims about reporting from memory. People recollect a fine-grain stimulus value with an associated inexactness and also may recollect a category, including boundary values, a prototype value, or both, each with an associated inexactness. The model posits two estimation procedures by which category information may be used to adjust a recollected fine-grain value in reporting. Next, we examine the consequences for reporting of using these estimation procedures at different locations in a category under varying conditions of inexactness.

To see why the use of category information may improve the overall accuracy of reports, consider a recollected fine-grain value near a category boundary. An inexactness is associated with the recollection. That is, there is a range of actual stimulus values that might have given rise to that recollection. For a stimulus recollected as being in the category, most of these possible values lie farther into the category. Hence, the proposed estimation procedures, which move reports farther into the category, may improve overall accuracy even while introducing bias. In formal terms, inaccuracy (i.e., the variability of reports around the true value) has two components: variability around the mean of reports (variance) and the difference between the true value and the mean of reports (bias). Even if bias increases, the accuracy of reports will improve if variability decreases by a greater amount.

Truncation resulting from boundaries. According to the model, people may adjust recollected fine-grain values to be consistent with a recollected category. That is, category boundaries constrain reports of fine-grain values to lie in the range subsumed by that category. Yet the distribution of inexactness for a stimulus near a boundary may extend past that boundary. Hence, a fine-grain value falling outside the category may be sampled. Such a value may then be adjusted, leading to a truncation of the underlying unbiased distribution of potential recollections. At inexact category boundaries, the effects of truncation may be averaged over the range of potential boundary values. The truncation of the memory distribution of fine-grain values will result in a shift of reports inward to the interior of the category (leading to bias, but decreasing the variance of the reports).

Bias effects resulting from truncation will be greater when fine-grain values are more inexact. Consider having to report the lengths of two objects that are 6 and 24 in., drawn from a set of objects known to range from 2 to 28 in. Assume that the long (24-in.) object is represented less exactly in memory than the short (6-in.) object. The two objects are equally far from boundaries, but a greater proportion of the distribution of uncertainty will be eliminated by truncation for a more inexactly represented value. Hence, bias for the long object resulting from truncation at the upper boundary (at 28 in.) will be greater than bias for the short object resulting from truncation at the lower boundary (at 2 in.).

Finally, bias effects resulting from truncation will be less for stimulus values near boundaries that are more inexact. When boundaries are inexact, not all stimuli falling in the range of the possible boundary values will have been encoded as lying in the

[1] William Goldstein suggested that the inexactness of recollected values might become accessible through sampling more than one value at the time of recollection and examining the variation in these values.

category. For stimuli that were not categorized at encoding, there will be no truncation. Hence, the average bias will be less than if all items were assigned to the same category.

Weighting with a prototype. The model holds that, as in Bayesian procedures, people may use assumptions about how instances are distributed (prototype and its inexactness) to adjust recollected fine-grain values.[2] In particular, people may adjust their reports using a combination of the recollected values at the two levels (fine-grain value and prototype), weighting them according to their associated inexactness. This process, analogous to regression to the mean, leads to a pattern of bias across the entire range of true values toward the prototype (but it decreases the variability of reports). The optimal weight of a prototype maximizes the decrease in variability in relation to the increase in bias. The extent to which variability can be reduced depends on the inexactness of the fine-grain value relative to the inexactness of the prototype.

If the inexactness of fine-grain values is equal across a category, the optimal prototype weight will be constant, leading to a linear pattern of bias toward the prototype. If inexactness changes over a dimension, the optimal weight of the prototype will be greater for more inexact values, and the pattern of bias may become noticeably nonlinear. Consider again our earlier example of two objects 6 and 24 in. long, where inexactness in representation is greater for the 24-in. object. If the set of objects is distributed around 15 in. (the prototype), bias for the long object should be greater than bias for the short object, because the prototype should optimally be given greater weight for the less exact long object.

Finally, bias effects resulting from weighting with a prototype, like those from truncation, will be less for stimulus values near boundaries that are more inexact. This is because when boundaries are more inexact, not all stimuli falling in the range of possible boundary values will have been encoded as being in the category. For those that are not so categorized, there will be no shrinkage toward the prototype. Hence, the average bias will be less for true values in the range of boundary uncertainty.

Category effects when fine-grain coding changes. Inexactness of fine-grain coding may increase across dimensions that increase in magnitude such as length, loudness, weight, and so on. In this case, bias resulting from truncation and prototype effects will be greater at larger magnitudes, leading to an overall downward bias in reporting. (Note that this is true of the example of the 6- and 24-in. objects referred to previously.) This bias is not due to distortions of fine-grain coding but rather to category effects that are larger when coding is less exact.

There is another way in which downward bias may arise for dimensions that increase in magnitude. When inexactness increases markedly along a dimension, people may use larger fine-grain units in coding stimuli (e.g., a longer object may be measured in feet rather than inches). Depending on where the shifts to larger measurement units occur, this may lead to a pattern of reports that diverges increasingly from physical values (downward bias). Huttenlocher, Hedges, and Bradburn (1990) found such a pattern of downward bias resulting from people's use of successively larger multiples in estimating the number of elapsed days since a target event (e.g., 7, 14, 30, and 60 days). If such larger units do not encompass (i.e., coexist with) a range of fine-grain values, they are not categories in the model; rather they are less refined fine-grain units.

Consider category effects when the units used in fine-grain coding increase in size across the region encompassed. Variability will be less for larger units (e.g., if a 16-in. object is always described as 12 in., i.e., 1 foot) than for smaller units (e.g., if a 16-in. object may be described as 10, 11, 12, . . . , 22 in.). Hence, category effects will be smaller when rounded values are used. Although the downward bias resulting from category effects (weighting with the prototype or truncation as a result of the upper boundary) will be smaller for rounded values, those effects will be superimposed on the downward bias arising from the shifts to successively larger rounded values (see Huttenlocher et al., 1990, for details).

Applying the Category Model to the Coding of Location

The literature on spatial memory provides evidence of bias in reports of item location. An obvious interpretation is that the representation of spatial location is distorted (i.e., that it does not correspond to objective measurement). However, the category model just referred to suggests an alternative: that item location is coded at two levels of detail, each of which is unbiased although inexact, and that bias arises in combining information from the two levels to produce an estimate.

Although existing studies show that simple metric models are inadequate, models that make precise predictions about the nature of bias effects have not appeared in the literature. Judgments of distance may be over- or underestimated (e.g., Baird, Merrill, & Tannenbaum, 1979; Baum & Jonides, 1979). They are affected by the presence of boundaries or barriers between items (Kosslyn, Pick, & Fariello, 1974; Newcombe & Liben, 1982; Thorndyke, 1981) and by the presence of reference points (Holyoak & Mah, 1982; Sadalla, Burroughs, & Staplin, 1980). Judgments of distance and orientation are affected when a space is hierarchically organized (i.e., involves more than one unit, neighborhood, state, and so on). Orientation judgments are distorted to reflect the relative orientation of the units (Stevens & Coupe, 1978). Distance judgments are similarly distorted, being under- or overestimated depending on whether the judged locations are in the same unit or in different units, respectively (MacNamara, 1986). Distortions of distance are found whether the boundaries between units are explicit or are only implicit (Hirtle & Jonides, 1985; MacNamara, Hardy, & Hirtle, 1989). These distance judgments correlate with the results of priming studies showing that the degree of association

[2] Note that the Bayes estimate for normal distributions of true values, when the distribution of errors is unbiased (and normal), is a linear combination of the mean (prototype) and the fine-grain value (recollection). Although biased, the Bayes estimate is more accurate than the recollection alone (see James & Stein, 1961). This remains true as the distribution of instances becomes less peaked (i.e., approximates a uniform distribution). Furthermore, shrinkage toward any of a large range of values interior to a category, not just the mean, increases accuracy (Deeley & Lindley, 1981). Hence, in a broad range of cases it may be optimal not to simply report a recollected fine-grain value, but rather to assign a nonzero weight to the prototype.

among items belonging to the same unit is greater than that among items in different units (MacNamara, 1986; Maki, 1981; Wilton, 1979). The latter result is supported also by greater amounts of clustering in free recall of items located within a unit (Hirtle & Jonides, 1985; MacNamara et al., 1989).

A study by Nelson and Chaiklin (1980) provided the impetus for applying our category model in the spatial domain. These investigators found bias in reporting location in a very simple case (a single dot within a circle). Their findings suggested to us that location in even the simplest of spaces may be represented at more than one level of detail. The simplicity of their task was attractive to us in that the complexity of the spaces explored in much of the existing spatial literature may have made it difficult to examine precisely the relations between spatial organization and response bias, even if these relations are simple. Nelson and Chaiklin presented a dot on a diameter line within a circle (at different angles on different trials). Then they removed the display and had subjects reproduce the location of the dot in a comparable display. In reporting errors, they divided the circle into an inner ring, a middle ring, and an outer ring, and examined assignments for dots with different actual locations. Errors were not symmetric around the true locations of the dots; rather there was a systematic tendency to misplace dots toward the circumference line, except near that line.

Nelson and Chaiklin (1980) proposed three postulates to explain the observed bias in dot placement: that the circumference serves as a landmark leading to accurate placement of dots proximal to it; that there is a systematic bias toward landmarks; and that the magnitude of the bias increases with distance from landmarks. The latter two postulates suggest that distances seem smaller for locations farther from the circumference; that is, portions of represented space are stretched or contracted relative to objective space. In fact, because Nelson and Chaiklin believed that mental representation may not correspond to objective measurement, they did not present metric information on errors.

Our model provides a more general explanation of the bias observed by Nelson and Chaiklin (1980) than their three postulates. We adopt their first postulate—that coding is more accurate near physically present reference points (the circumference)—but not their other two postulates. Instead, we posit that dot location is coded at two levels of detail; a fine-grain location (an idealized point) and a category (a radial segment). Bias arises from combining information at the two levels in making a response. In particular, we hypothesize that the fine-grain value is weighted with a prototype central to the radial segment in estimating dot location. Bias resulting from the prototype increases with uncertainty of fine-grain coding. Hence, there should be considerable outward bias for dots near the center where location coding is inexact and a small amount of inward bias for dots near the circumference where location coding is precise. Nelson and Chaiklin's use of nonmetric data may not have been sensitive enough to uncover such a trend. Our model makes use of metric information to predict the size as well as the direction of errors. Furthermore, our model is not restricted to explaining bias in just one dimension (along a diameter line), but can predict bias in reports of dot location in a homogeneous space, as indicated next.

Coding Location in a Circle

In our studies, subjects are presented with a single dot in a homogeneous circle. Subjects respond by indicating the location of the dot in a comparable circle (as opposed to Nelson & Chaiklin's, 1980, circle with a diameter line). Two dimensions are required for coding location. We posit that coding on each dimension is made at two levels of detail.

Coding fine-grain location. We assume that coding location by eye is analogous to the physical measurement of location. Either of two conventional coordinate systems can be used. One involves a distance and an angle (e.g., polar coordinates), and the other involves two distances along a rectangular grid (e.g., Euclidean coordinates). In polar coordinates, a dot can be located by imputing a line that is the shortest distance to the circumference. The length of this line specifies radial location. The line is perpendicular to the circumference and, if extended, would pass through the center of the circle. The orientation of this line relative to the horizontal or vertical axis specifies angular location. In rectangular coordinates, a dot can be located by imputing two lines, parallel to the horizontal and vertical axes, respectively, between the dot and the circumference (each in the shorter direction). Specification of location on each dimension has two components: the distance to the circumference and the location where the imputed line crosses the circumference. We also assume, as did Nelson and Chaiklin (1980), that inexactness of coding will increase as the distance of the dot from the physically present reference (the circumference line) increases.

Error patterns at different locations will depend on which coordinate system subjects use. Thus, examination of graphic displays of responses at different locations should allow us to determine which coordinates subjects use. Consider the shapes of graphic displays of responses at different locations if polar coordinates are used. For radial location, inexactness of coding will increase with distance from the physically present circumference line at all angular locations. Hence, graphic displays will show a distribution of radial values that is narrow near the circumference and increasingly wider toward the center. For angular location, inexactness of coding may be constant across angular locations, because there is no physically present reference for measuring angle. As radial locations approach the circumference, distances between adjacent angular locations increase, so graphic displays will be increasingly stretched out along an arc parallel to the circumference. (See displays in Figure 1A, which show the outline of hypothetical distributions of reports at two locations.)

Consider the shapes of graphic displays if subjects use rectangular coordinates. Again, inexactness of coding increases with distance from the physically present circumference line. Hence, graphic displays will be elliptical at all locations where the x and y distances differ, with axes of symmetry that are parallel to the horizontal and vertical axes. There will be greater dispersion in the direction of greater uncertainty. On the diagonals, the displays should be circular, and should increase in size with distance from the circumference. (See displays in Figure 1B, which show the outline of hypothetical distributions of reports at two locations.)

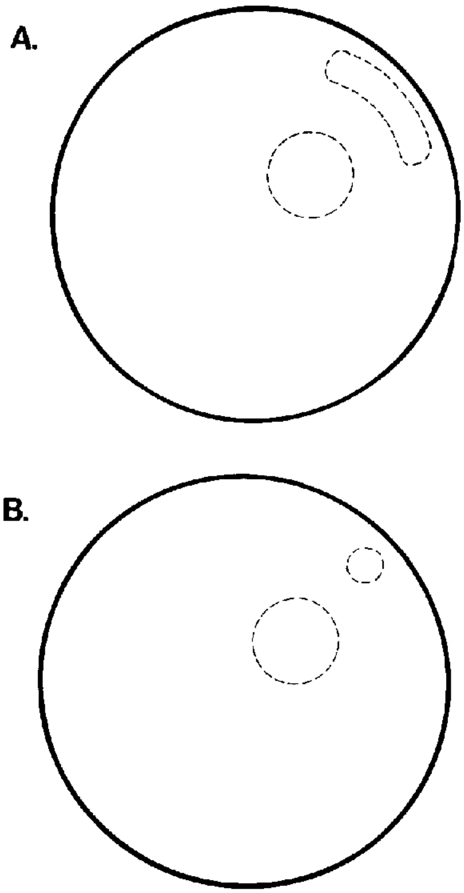

Figure 1. Outlines of hypothetical distributions for polar coordinates (A) and rectangular coordinates (B).

In addition, if values for the two dimensions are obtained independently when measuring by eye as when physically measuring location, errors in coding on the two dimensions should be statistically independent in the coordinate system subjects use. We investigate mathematically whether the two dimensions are independent in the following way. We determine the correlation between errors at each actual location (i.e., observed location minus actual location). We test whether these correlations are all zero by obtaining the average correlation across all locations and by examining the variability of those correlations. If measurement for the two dimensions is independent, the average correlation should be nonsignificant, and the variability of the correlation should be small.

Coding coarse-grain location. Coarse-grain coding breaks up dimensions into regions (categories). Before exposure to a set of instances (i.e., dots shown in particular locations), categories may be formed on the basis of the perceptual organization of the figure itself or its orientation in a larger field. For a circle, where all diameters are axes of symmetry, subdivision of the figure depends on its orientation in a larger field. An accessible basis for coarse-grain coding would be to orient the circle relative to the self. The body axes may be used in imposing on the circle horizontal and vertical axes, which cross at its center. Let us assume that subjects use polar coordinates (thus anticipating our findings). In this case, the imputed horizontal and vertical axes lead to radial categories, each extending between the circumference and the center, and to angular categories, each extending across a 90-degree range of angles. Radial prototypes arising from such perceptually based categories might lie on an imputed circle, which divides the actual circle into two equal areas (approximately two thirds of the way from the center to the circumference), and angular prototypes might lie in the vicinity of the diagonals.

After exposure to a set of instances, spatial categories based on perceptual organization may not be maintained. Recall that the model holds that a function of categories is their potential for improving estimates of particular values. For certain distributions of instances, perceptually based categories may not be the best for this purpose. For example, if the instances presented are unevenly distributed, with a concentration near the horizontal and central axes, categories with boundaries at the diagonals and prototypes at the horizontal and vertical axes would be more effective. Here we use approximately uniform distributions of instances. This is because our purpose is to assess the category model, including the prototype mechanism, not to explore how varying conditions may affect the locations of category boundaries and prototypes.

Estimates of Location

Evidence that categories are used in estimation consists of a pattern of bias away from boundaries. In the present case, the boundaries lie along the horizontal and vertical axes and at the center of the circle (where the axes cross). For locations sufficiently near the boundaries, the distribution of reports of angular and radial location will be truncated. The magnitude of these bias effects decreases rapidly with distance from the boundary (as discussed in the Appendix and as shown in Figure A2). These effects are negligible when the distance from the boundary is two standard deviations or greater. To anticipate, truncation is not a major source of bias in locating a dot in a circle.

Use of a prototype leads to bias across the entire range of values toward a value interior to the category. The optimal weight of the prototype depends on the relative inexactness of the particular remembered value and the prototype. The inexactness of the prototype should be constant across instances because the distribution of instances is approximately uniform. The inexactness of particular values may vary for either of two reasons. First, fine-grain values may become inexact as a result of loss from memory. If other activities intervene before a response is made, inexactness of representation of particular locations should increase, and greater weight should be given to angular and radial prototypes. Second, we assume that the representation of fine-grain values becomes more inexact as magnitude (i.e., distance from a fixed reference) increases. Hence, a radial prototype should have somewhat greater weight near the center than near the circumference (although this is a minor factor in our small circle). Finally, consider prototype effects in the region of quadrant boundaries. Because stimuli sufficiently close to an inexact boundary sometimes will not be correctly categorized, bias should decrease very near the horizontal and vertical axes.

We have argued that the use of a prototype can improve the accuracy of reports. Hence, we consider the conditions under which a prototype would improve estimates of angular or radial location under the assumption of a uniform distribution of locations, and of equal (or roughly equal) inexactness for fine-grain values at different locations. Consider first when use of an angular prototype will improve accuracy of reports. The variance of a uniform distribution with range a is $a^2/12$.[3] The angular range for a quadrant in the circle is $\sigma_A{}^2 = 90^2/12 = 675$ or $\sigma_A = 26$. Thus, if the variance of subjects' recollections from memory for angular location is greater than this, shrinkage toward a prototype will improve estimation. Also consider when use of a radial prototype will improve accuracy of reports. The range of radial locations in our numbering scheme is 0 to 1; hence, the variance of radial locations is $\sigma_R{}^2 = 1/12 = .083$ or $\sigma_R = .29$. Thus, if the variance of subjects' recollections from memory for radial location is greater than this, shrinkage toward a prototype will improve estimation.

Testing the Model

We present four experiments in which people place a dot in a homogeneous circle. In Experiments 1 and 2, we demonstrate the use of polar coordinates and the organization of the circle into quadrants with a prototypical angular value and radial value in each quadrant. Having shown that the pattern of bias in reports of location is generally consistent with the predictions of the category model, we test the formal model, which explicitly incorporates prototypes, truncation as a result of boundaries, and uncertain boundaries in Experiments 3 and 4. In Experiment 3, we assess the predictions of the model that uncertain boundaries will lead to decreased bias near boundaries by obtaining data for angular locations close to the horizontal and vertical axes. In Experiment 4, we obtain data on biases in dot placement after an interference task. This allows us to assess the prediction that increasing the inexactness of the representation of particular values will increase the slope of the bias curve without affecting other aspects of its shape because greater weight is assigned to the prototype.

Experiment 1

In the first experiment, we make a preliminary evaluation of the coordinate system subjects use and of the proposed categorization model. We show that the pattern of variability of responses across locations is that predicted for polar coordinates, and these two coordinates are independent of one another. We also show that item location is represented at two levels (particular value and category). There is radial bias away from the center in a homogeneous circle (as there was in the circle with a diameter line studied by Nelson & Chaiklin, 1980). There is angular bias away from the horizontal and vertical axes extending across the range of values toward a prototype near the center of each quadrant. Finally, in this task, we show that use of a prototype improves the overall accuracy of estimation of dot location.

Method

Subjects. Subjects were 50 University of Chicago students and staff between the ages of 17 and 35 years, drawn from a list, maintained by the Department of Psychology, of people interested in participating as subjects in experiments.

Materials. The stimuli consisted of 69 8.5 × 11-in. (22 × 28 cm) white sheets of paper. Each sheet had a 15-cm circle printed on it in black. A black 1.5-mm dot was printed within the circular figure on each page. No dot position was repeated in the stimulus set. The 69 dots were assigned positions uniformly distributed over the circle. The positions taken together made up a grid with rows and columns positioned 16 mm apart, a single dot at each of the interstices. Seventeen dot positions fell on the center horizontal and vertical diameters of the circle: 1 dot exactly at the center of the circle and 4 at each of four radial distances between the center and border.

Stimulus sheets were presented in a booklet with steel clip-on document rings on the upper edge. The sheets were separated by card-weight blank sheets. The booklet was suspended over an easel so that subjects viewed the stimuli at about a 25-degree angular cant from the vertical axis. Responses were made on sheets of paper laid flat on the table between the subject and the apparatus. Stimuli were presented in one of five random orders. Response sheets were identical to stimulus sheets with the exception that the circles contained no dot.

Procedure. Subjects participated individually. The experimenter sat next to the subject. The response sheets were positioned 9 in. (22.86 cm) from the table's edge. Subjects were told that we were "interested in finding out how accurately people can reproduce the locations of dots that they see." Each randomized stimulus set was shown to 10 subjects. As in Nelson and Chaiklin's (1980) study, the stimulus was presented for 1 s, followed by an 8-s intertrial interval, timed with the aid of a tape recording to which the experimenter listened on earphones.

On each trial the experimenter placed a response sheet in front of the subject and then said "now," which was the signal to look at the display apparatus. Two seconds later the stimulus was displayed by lifting the cover sheet for 1 s. The subject's writing hand was kept in the lap until after the stimulus was covered. Then the subject marked the location of the to-be-remembered dot on the response sheet. The stimulus sheet, together with its mask, was then flipped to the back side of the display apparatus. After responding, the subject placed the response sheet face down on the table next to the response area, and a new response sheet was placed in front of the subject. Five practice trials were given.

Responses were measured using a coding grid consisting of a 15-cm circle on a transparency with a grid marked off in 1.5-mm units, or 100 × 100 square units. Each response sheet was coded by laying the transparency on top of it, bringing the two into alignment, and noting the x and y coordinates of the subject's dot. Reliability was checked with independent scoring by a second person, and discrepancies were resolved by rechecking.

Results

Rectangular versus polar coordinates. We first examined scatter plots of subjects' responses for each of the 69 dots. Figure 2 shows the response distributions for three selected dots. In general, as can be seen for Plots A and B, the distribution of responses for dots farther from the center are more elongated and parallel to the circumference, as would be expected. As can be seen for Plots B and C, the distribution of responses is similar at very different angular locations, which are equal in distance from the center of the circle. This distribution corresponds to what one would expect if subjects were using polar

[3] This is because the variance of a uniform distribution that ranges between 0 and 1 is known to be 1/12 (see Johnson & Kotz, 1970). Hence, the variance of a uniform distribution that ranges between 0 and a is $a^2(1/12)$.

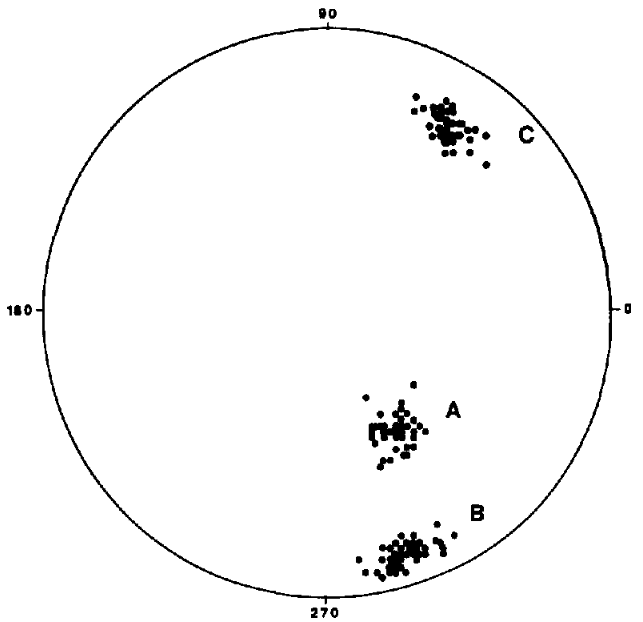

Figure 2. Graphic displays of responses for dots at three locations in Experiment 1.

coordinates. Furthermore, polar coordinates are consistent with subjects' informal descriptions of dot locations in terms of angle ("it was about 11 o'clock") and distance from the center ("it was about two thirds of the way out"). For these reasons, we converted the data to polar coordinates.

Error in angular location was determined for each response by subtracting the actual angular location value of each stimulus dot from the response value. Radial error was determined in an analogous fashion. Most of the subjects' responses formed an identifiable and reasonably compact distribution. However, a few of the responses were so far off as to suggest that the subject either forgot or failed to encode the dot location. In fact, subjects occasionally commented that they forgot the dot's location. We used two procedures to remove such responses from the data set. First, any response dot that was more than 45 degrees away in either direction from the angular location of the stimulus dot was removed (2.4% of the responses). Second, response dots that fell more than three standard deviations from the mean of the distribution of either angular or radial location also were deleted (an additional 1.5% of the responses). We replicated our analyses using the "noisier," unculled data set here and in all later experiments to ensure that these deletions did not alter the overall pattern of responding. In no case did they do so.

Variability of reports of angle and radius across locations. The standard deviations of reports of radial location are largest near the center of the circle and decrease monotonically toward the perimeter of the circle ($r = .59$, $p < .01$). In contrast, the standard deviations of reports of angular location are independent of actual angular location. The correlation between angular standard deviation and angular location for points interior to the quadrants is $r = .01$ (not significant). These are the predicted patterns if people use polar coordinates. It should be noted that standard deviations of reports of angular location at the horizontal and vertical axes were in the same range as at other locations.

Independence of dimensions: Errors in angular and radial location. Next consider whether radial and angular location are independent features of coding. This question was investigated by computing a correlation coefficient between angular error and radial error at each location and examining the distribution of these correlations across locations. First, the average correlation ($r = -.02$) was not significantly different from zero. Second, although there was a statistically significant amount of variability in correlations across locations as measured by a variance component estimate (see Hedges & Olkin, 1985), the absolute magnitude of this variability was quite small (a variance component of .015). To anticipate, similar analyses performed for Experiments 2, 3, and 4 also show that angle and radius are independent dimensions. Hence, the dimensions are examined separately in this article.

Bias in angular reports. The dots that are located directly on the vertical and horizontal axes at $\theta = 90$ degrees, 180 degrees, 270 degrees, or 360 degrees showed little angular bias, especially on the vertical axis. For the vertical axes (at 90 degrees and 270 degrees), the mean of reports differed from the actual by 0.9 and −0.2 degrees, respectively. For the horizontal axes (at 180 degrees and 360 degrees), the mean biases were 2.2 and 5.9 degrees, respectively. In contrast, bias for those points close to but not on the axes is between 8 and 10 degrees.

Because the variability of reports was constant across locations, the relation between bias and actual angular location should be linear. Figure 3 shows the mean of the angular response errors for each stimulus dot plotted against the actual angular location of the stimulus. The figure shows that essentially the same pattern is repeated within each of the four quadrants. (For ease of reference, we refer to the quadrants as I, II, III, and IV, beginning with the upper right and progressing counterclockwise around the circle.) Within each quadrant there is a strong linear relation between angular error and actual angle ($r = -.95, -.96, -.88$, and $-.97$ in Quadrants I, II, III, and IV, respectively, each $p < .001$). The discontinuous function in Figure 3 indicates that, within each quadrant, estimates are biased away from the actual dot locations in a direction toward the center of each quadrant. The slopes of the empirical regression lines of angular bias on actual angle are also similar in each quadrant ($b = -.16, -.18, -.20$, and $-.20$ in Quadrants I, II, III, and IV, respectively). In each case the regression line intersects zero near the angle corresponding to the middle of the quadrant. This relation implies that within each quadrant responses are biased away from the horizontal and vertical axes and toward a neutral point near the angular center of the quadrant.

The neutral (prototypic) angular values estimated from the within-quadrant regressions were at $\theta = 61$ degrees, 142 degrees, 235 degrees, and 311 degrees, respectively, quite close to numerical angular centers of the quadrants located at 45 degrees, 135 degrees, 225 degrees, and 315 degrees, respectively. Only in Quadrant I does the neutral point estimated from subjects' response dots differ significantly ($p < .05$) from the angular center of the quadrant. This pattern of results arises in later experiments also; the prototype in the upper right quadrant is not at the actual middlemost value. Although our concern here is with subjects' use of prototypes rather than with the particular value

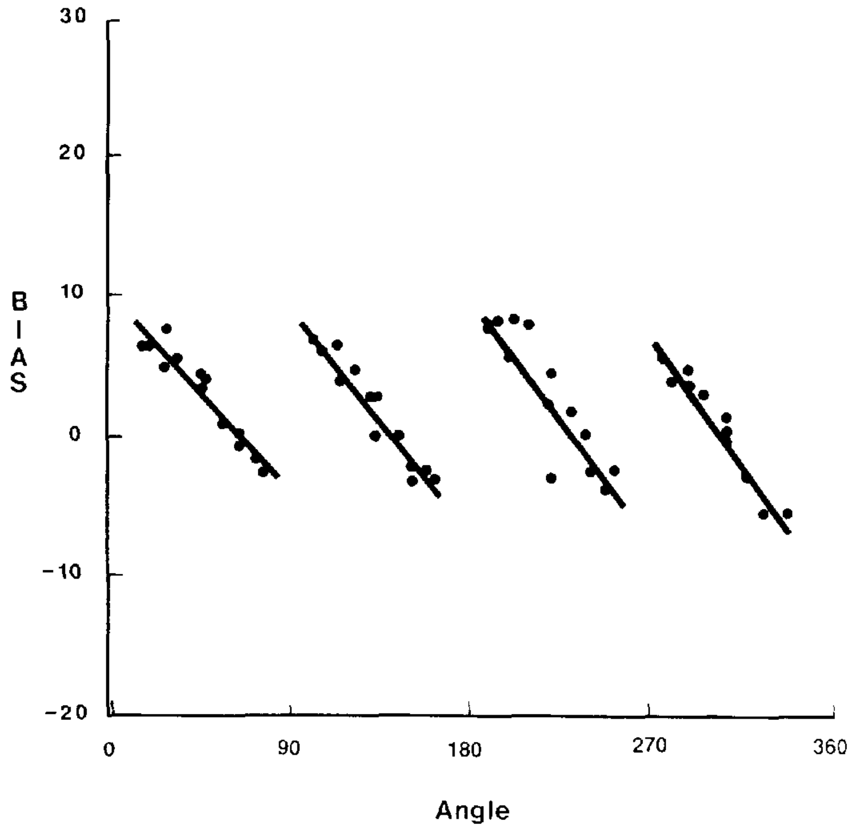

Figure 3. Mean angular bias in responses as a function of stimulus location for Experiment 1. (Solid lines are within-quadrant regression lines of angular bias on angular location.)

of those prototypes, it should be noted that this upper right quadrant effect does not depend on how the circle is presented or on the mode of response, because these varied across experiments.

The strong linear relation between bias and actual location depicted in Figure 3 provides strong support for the hypothesis that subjects use prototypes. The uncertainty of reports is too small to produce substantial bias from truncation at quadrant boundaries even for those dots nearest the boundary. That is, the actual angular locations of points not on the axes are more than two standard deviations (of the distribution of reports for a single dot) from the axes, so truncation at the boundaries will necessarily be negligible.

Bias in radial reports. The uncertainty of reports is largest near the center of the circle and decreases toward the circumference. Hence, bias will be greater near the center. Figure 4 shows the bias in reports of radial location. The mean error in reports of radial location for each stimulus dot is plotted against the actual radial location of the dot. Radial distances have been scaled to reflect the proportion of the distance between the center of the circle and the circumference. Hence, $r = 0$ at the center of the circle and $r = 1.0$ at the circumference. The plot shows that reports for points located near the center of the circle exhibit substantial bias in the outward direction, whereas reports for points near the circumference of the circle show no obvious bias.

The location of the prototype is at the point of zero bias within a category. Thus, we identify the prototype as the point at which the bias curve crosses zero. Here and in later experiments, we examined the zero points estimated from a series of nonlinear approximations and a linear fit and determined that they did not differ appreciably. Consequently, we used the simpler linear approximation to estimate prototype location. The slope of the empirical regression line of radial bias on actual radius is $b = -.066$, $p < .01$. The estimate of the neutral point thus obtained was at a radius of .91 with a 95% confidence interval (computed using Feiller's theorem) of .88 to .94. This value is sufficiently close to the circumference so that there is no possibility of detecting inward bias toward the prototype. A possible explanation of the location of the radial prototype is that, because subjects used polar coordinates, their notion of a uniform distribution may be of a distribution that is uniform in radius and angle. Such a distribution would have fewer dots near the periphery than a distribution that is uniform in rectangular coordinates. Thus, the distribution we presented appears more densely concentrated toward the periphery than toward the center of the circle. This can be seen clearly in Figure 4, which shows that the largest concentration of dots is at radii of at least .8. Furthermore, the set of points presented included some points that were extremely close to the circumference line. Therefore, it would not be surprising if subjects formed a prototype lying near the circumference.

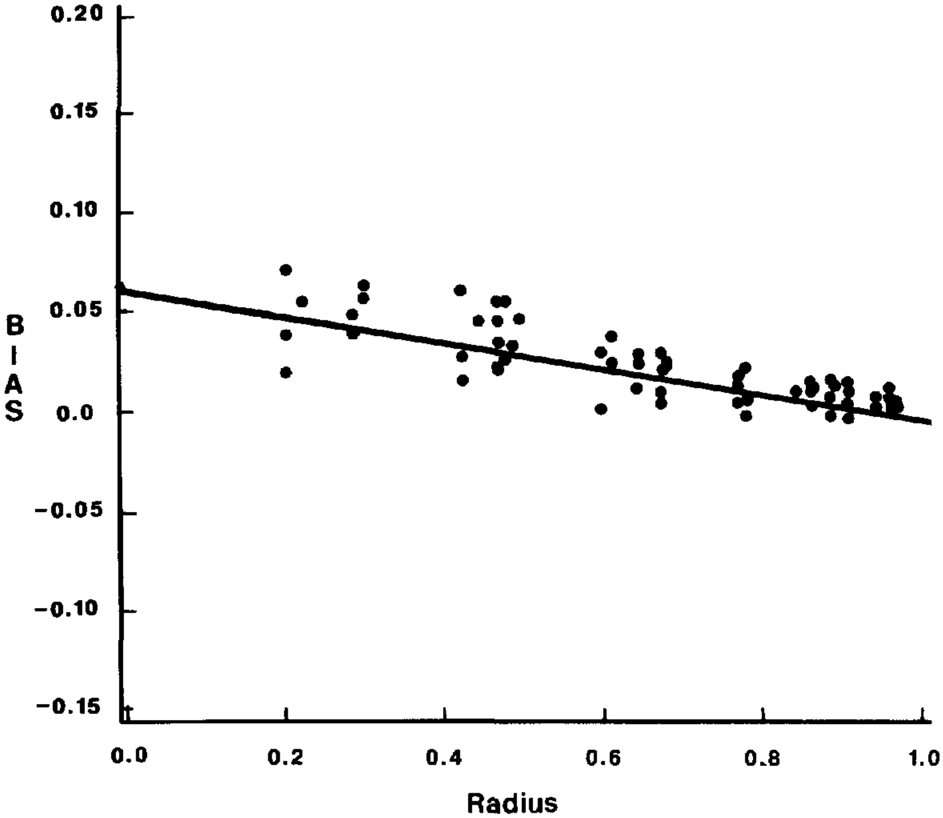

Figure 4. Mean angular bias in responses as a function of stimulus location for Experiment 1. (The solid line is the regression of radial bias on radial location.)

Advantages of angular and radial prototypes. Typical standard deviations of reports of angular location were $\sigma_R = 10$ or a variance of $\sigma_R^2 = 100$. However, the reports exhibited shrinkage toward the prototype as evidenced by the slope λ of the regression of angular bias on angular location. The variance of the report is λ^2 times the variance of the recollections from memory σ_M^2. Thus, $\sigma_M^2 = \sigma_R^2/\lambda^2$. Given a typical shrinkage (slope) coefficient of $\lambda = .2$ and a typical value of σ_R^2, we compute that a typical value of σ_M^2 is $\sigma_M^2 = 100/(.2)^2 = 2{,}500$. Because this is much larger than $\sigma_A^2 = 675$, the variance of the distribution of true angular locations, shrinkage toward angular prototypes in each quadrant should produce a more accurate report than one based on fine-grain recollection alone. The values of the observed slope of radial reports as a function of actual radius is $\lambda = -.066$, and values of the variance of observed reports of radial standard derivation are at least .025, which implies a standard derivation of uncertainty on recollections of at least $(.025)^2/(.07)^2 = .13$. Because this is much larger than the variance of actual radial locations $\sigma_R^2 = .083$, the use of the radial prototype should produce a more accurate report than one based on fine-grain recollection alone.

Experiment 2

In the second experiment, we present a distribution of dots that is more nearly uniform in polar coordinates, because the analysis of the data in Experiment 1 indicated that a polar coordinate system more nearly described the way subjects estimated dot locations. One purpose is to determine whether this will result in a radial prototype farther from the circumference than in Experiment 1. In addition, to obtain more systematic data at certain angular and radial locations, we include a set of dots on the diagonals. Although there was no angular bias at 45 degrees in Experiment 1 except in one quadrant (items are moved from both directions toward 45 degrees), only a few dots were positioned at approximately 45 degrees. In addition, graphic displays on the diagonals will provide further evidence as to the use of polar coordinates, because these displays should be strikingly different in the two coordinate systems. Distances along the two rectangular coordinates are equal on the diagonals, so the displays should be circular for all radial locations. The sizes of these circular displays should be smaller near the circumference. In contrast, for polar coordinates, the displays along the 45-degree axes should be more elongated along an axis parallel to and near the circumference.

Method

Subjects. The 25 subjects in this study were selected in the same way as those in Experiment 1.

Materials. A total of 128 dots was presented. A basic set of 80 dots assigned using polar coordinates was included. In addition, 48 filler dots lying at intermediate positions were added to create a more uniform density of dots over the surface of the circle. This was because when we viewed the basic set of 80 dots in an ensemble, the distribution of locations looked so nonuniform that we thought the subjects might notice the differences in density. The basic 80 dots plus the filler dots represent a compromise set, intermediate between uniformity in polar coordinates and uniformity in rectangular coordinates.

Consider first the basic set of 80 dots. These were placed at 20 different angular distances from zero around the circle. Within each quadrant of the circle, five angular locations were selected as follows: 8 degrees in toward the center of the quadrant from each of the two axes bordering it, 26.5 degrees in from each of the axes, and the 45-degree angle within the quadrant. In choosing dots near the major axes, we obtained pilot data that indicated that angular distance must be 8 degrees from a horizontal or vertical axis for subjects to reliably determine in which quadrant a dot was positioned. The remaining angular positions in each quadrant were set exactly halfway between 8 degrees and 45 degrees. Four radial positions were used based on proportions of the distance between circle center and the perimeter, at .3, .5, .7, and .9 of the way from center to perimeter. The pilot data indicated that radial distances must be .3 of the way from center to perimeter for subjects to reliably specify the quadrant. The other radii were chosen so as to cover the distance to the circumference line.

Forty-eight filler locations increased the uniformity of dot locations over the circle. Twelve locations were added to each quadrant, four at a radial distance of .6, and eight at .8 of the way between the center and the perimeter. The angular locations for the four .6 locations were 17.0 degrees and 35.5 degrees toward quadrant center from each of the border axes. For the eight .8 locations, the angular locations were 14 degrees, 20.5 degrees, 32.5 degrees, and 39 degrees in from each of the axes.

Procedure. Both presentation of stimuli and collection of subjects' responses were automated through the use of a Zenith microcomputer with a graphics cathode-ray tube (CRT) and a digitizing tablet. The 128 stimulus dots were each presented within a 15-cm circle on a Zenith flat screen 14-in. graphics monitor (ZCM-1490). Because the monitor's image resolution was insufficient to present a perfectly rounded circle, we produced the stimulus circle on a transparent acetate, which was superimposed on the monitor screen. Subjects viewed the stimulus dot within this overlay circle. The monitor was in inverse video mode, so the dot appeared dark on a light background. The monitor was positioned so that the center of the stimulus circle was just an inch or two below subjects' eye level and about 2½ ft away.

Responses were collected using a Scriptel high-resolution, clear-glass digitizing pad on which the subjects responded with a stylus. A 15-cm circle printed on a white background was positioned under the clear pad. Subjects responded by using the stylus to indicate locations within this circle. The Scriptel stylus is held like a pencil. A small tip at the end, when depressed against the digitizing pad, magnetically signals the coordinates of the indicated location to the computer, which then stores these responses (paired with the actual locations of the stimulus dots). The pad was positioned on a response surface, which slanted down toward the subject at an angular cant of approximately 25 degrees from the horizontal axis. The top edge of the pad was 5 in. below the CRT screen. Subjects were instructed to hold the stylus down at their sides between trials. The instructions described the purpose of the experiment and the importance of accuracy, as in Experiment 1.

The presentation intervals and practice trials were identical to those in Experiment 1. Order of presentation was randomized for each subject. A short beep signaled that a dot was about to appear. Each dot was on screen for 1 s, during which time the subject's writing hand was kept at his or her side. After the dot disappeared, the subject indicated its location on the response circle. If for any reason the subject failed to respond in the allotted time, the dot was presented again on a subsequent trial. Otherwise, subjects saw each dot location only once.

Results

The graphic displays in Figure 5 show two points on one of the diagonals. The shapes of these displays show clearly that subjects use polar coordinates. If rectangular coordinates are used, both the outer and the inner displays should be circular, and the outer display should be the smaller one. In contrast, if polar coordinates are used, the outer display should be parallel to the circumference; it should be more spread out along the circumference than the inner display (but less spread out in terms of radial values). Clearly, these displays are as expected for polar coordinates.

As in Experiment 1, analyses were performed after extreme outliers from the distributions of estimates for each dot were culled from the data set. This resulted in removal of 2% of responses. In addition, .9% of responses that fell more than three standard deviations from the mean of either angular or radial location also were deleted.

Independence of dimensions. As in Experiment 1, radius and angle proved to be independent features of coding. The average amount of angular bias bears no systematic relation to the amount of radial bias. The average correlation between angular error and radial error for Experiment 2 was not statistically significant ($r = -.01$), and the variance component measuring variation of correlations across locations was quite small (.013).

Bias in angular reports. The pattern of bias in angular error for Experiment 2 is the same as in Experiment 1. As in Experiment 1, the standard deviation of reports of angular location appears to be independent of actual angular location ($r = .15$, not significant). Consequently, the relation between angular bias and actual angle would be expected to be linear within quadrants. The plot of mean angular error against actual angle in Figure 6 again shows that, within each quadrant, responses

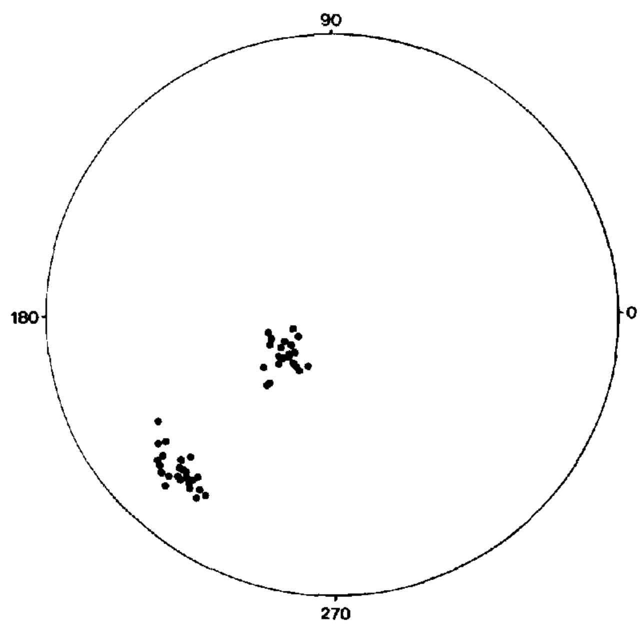

Figure 5. Graphic displays of responses for dots on a diagonal in Experiment 2.

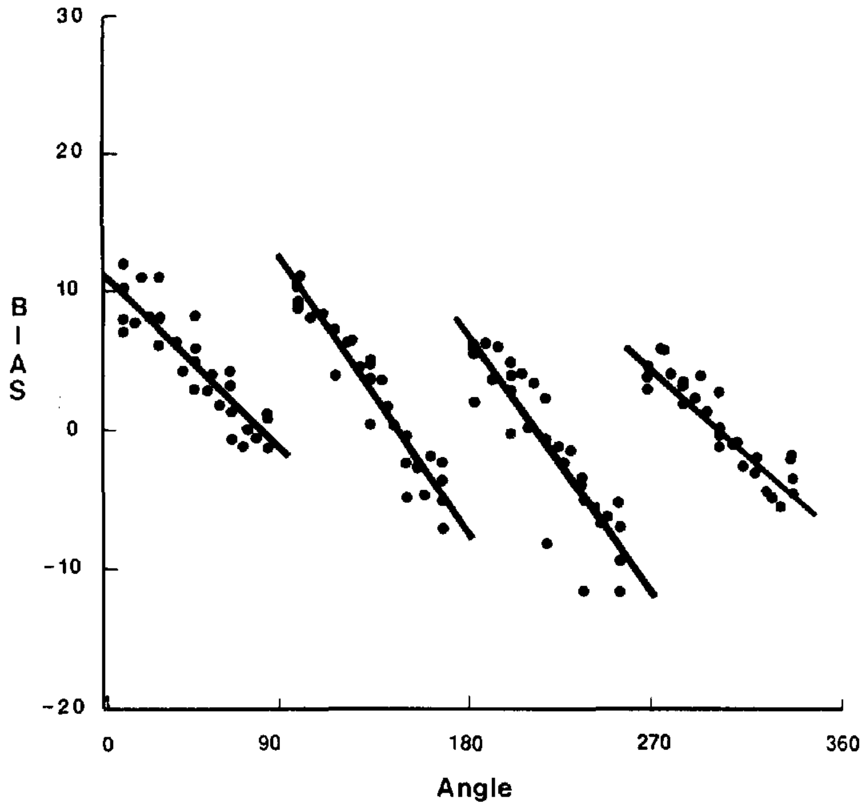

Figure 6. Mean angular bias in responses as a function of stimulus location for Experiment 2. (The solid lines are within-quadrant regression lines of angular bias on angular location.)

for dots located within quadrants are biased away from the horizontal and vertical axes and toward a point near the angular center of the quadrant. As in Experiment 1, the linear relation is very strong in each quadrant ($r = -.90, -.96, -.88$, and $-.92$ in Quadrants I, II, III, and IV, respectively). The slopes of the empirical regression lines are quite similar to those found in Experiment 1 ($b = -.14, -.20, -.19$, and $-.12$ in Quadrants I, II, III, and IV, respectively). In each quadrant, the regression line intersects zero in the regions of the angular centers of the quadrants. The estimated points of zero bias are $\theta = 73$ degrees, 145 degrees, 217 degrees, and 313 degrees, respectively. As in Experiment 1, only in the first quadrant was the prototype value significantly different from the angular center ($p < .05$).

Bias in radial reports. The standard deviation of reports of radial location was even more strongly related to radial location than in Experiment 1 ($r = -.92$, $p < .01$). The plot in Figure 7 shows that reports about points located near the center of the circle are substantially biased in an outward direction, whereas reports about points near the circumference are slightly but clearly biased inward. As discussed previously here, the relation between actual radius and bias in reports of radial location is not strictly linear. Yet we use a linear fit to estimate the location of the radial prototype. The slope of the empirical regression line of radial bias on actual radius is $b = -.05$, $p < .01$, and the point of zero bias estimated using a linear approximation is .66.

It should be noted that because the dots were shown on a CRT screen and the circle was an acetate overlay, the dots appeared somewhat behind the circle. It is at least possible that dots near the circumference line were judged to be farther away from that line than if they had been on the same surface. Hence, there could be greater inward bias and a lower estimate of the neutral point than would otherwise be found. Thus, more data on radial prototypes are needed.

Advantages of angular and radial prototypes. As in Experiment 1, the use of the prototypes improved the accuracy of reports of angular location. The typical standard derivation of angular reports of $\sigma_R = 10$ and typical slopes of bias on actual angular locations of $\lambda = .2$ once again implied variance recollections for memory of angular location on the order of 2,500, a much greater value than the variance of actual angular locations. Similarly, the typical standard deviation of radial reports was $\sigma_R = .025$, and the slope of radial bias on radial location was $\lambda = -.06$, which implied a variance for recollections of radial location on the order of .13, much greater than .08, the variance of actual radial values. Hence, the use of angular and radial prototypes improved the accuracy of reports.

Experiment 3

The third experiment was designed to determine the preciseness of the subjective quadrant boundaries by introducing dots

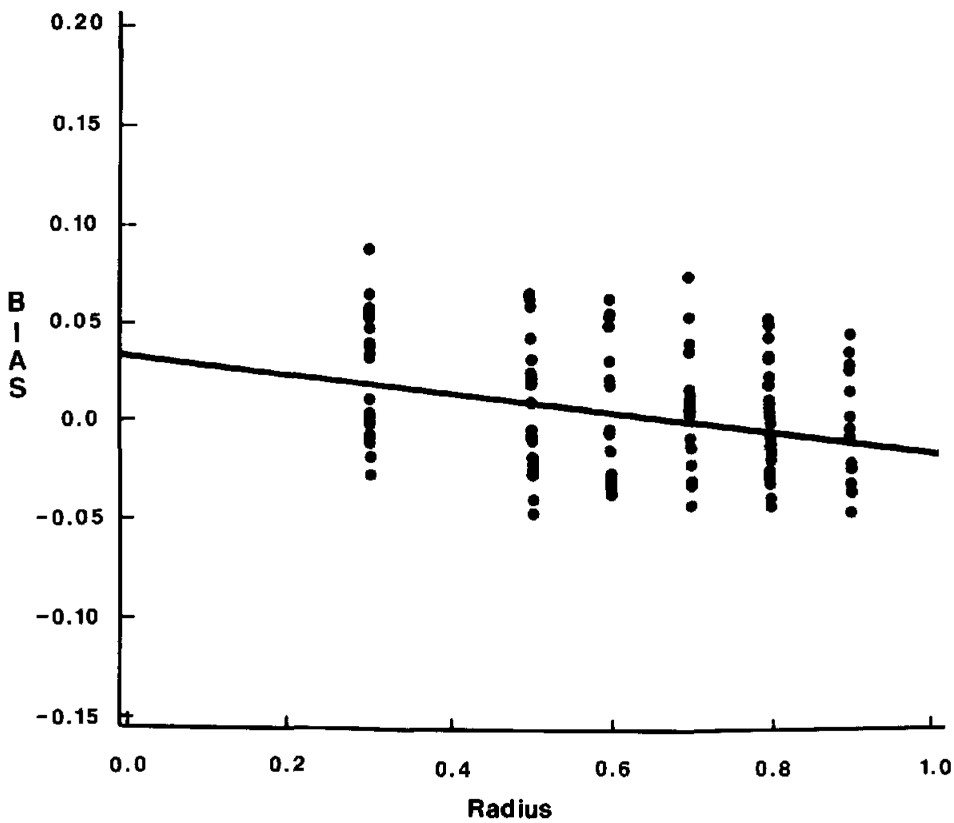

Figure 7. Mean radial bias in responses as a function of stimulus location in Experiment 2. (The solid line is the regression of radial bias on radial location.)

with locations close to the boundaries. The pattern of angular bias across the quadrant is modeled to determine whether the location of the prototype and steepness of the slope are similar to those in Experiments 1 and 2, except near the boundaries. In addition, we examine further the radial prototype using a presentation where the circle appears in the same plane as the dots.

Method

Subjects. The 25 subjects in this study were selected in the same way as those in Experiments 1 and 2.

Materials. The stimuli consisted of 112 8.5 × 11-in. sheets, each with a circle and dot printed on it, as in Experiment 1. The stimulus dots were positioned as follows: 28 angular positions around the circle were selected, 7 in each quadrant. Dots were positioned on the 45-degree angle within each quadrant and at three successive intervals of 13 degrees away from 45 degrees in either direction. Thus, the theta values within the quadrant were as follows: 6 degrees toward the center of the quadrant from each of the two axes bordering it, 19 degrees in from each of the axes, 32 degrees in from each of the axes, and the 45-degree angle in the middle. Four radial positions were selected to yield an average radial value of .5. A dot was positioned at .2, .4, .6, and .8 of the way from center to perimeter on each angular location.

Five copies of the original stimulus set were created, and the order of stimulus sheets within each set was randomized. The digitizing pad lay flat on the table in front of the subject, a response surface similar to that used in Experiment 1. The stimulus display apparatus was on the surface of the table on the other side of the digitizing pad from the subject.

Procedure. The methods and procedures were the same as in Experiment 1 except for use of the digitizing pad.

Results

As expected, our procedure for culling outlying responses (more than 45 degrees from the true angular value) resulted in the removal of more responses than in the first two experiments (5.2%). Recall that radial values of .2 were included in this study; this value is near the center of the circle, where small absolute errors of radial location create large errors of angular location. When a dot is misplaced to the other side of the center, angular errors in the order of 180 degrees result. This indeterminacy of angular location very near the center of the circle is a limitation of the polar coordinate system. The greater frequency of outliers that were culled (i.e., about 2% more than in Experiments 1 and 2) is entirely attributable to dots whose actual radial locations are .2. The errors, as expected, were in the order of 180 degrees; such reports did not occur in Experiments 1 and 2. An additional 1.2% of responses were eliminated because they fell outside the range of three standard deviations in angle or radius.

Independence of dimensions. The amount of angular bias bears no systematic relation to amount of radial bias. The average correlation between angular and radial bias was $r = .04$ (not significant) and the variance component of correlations across

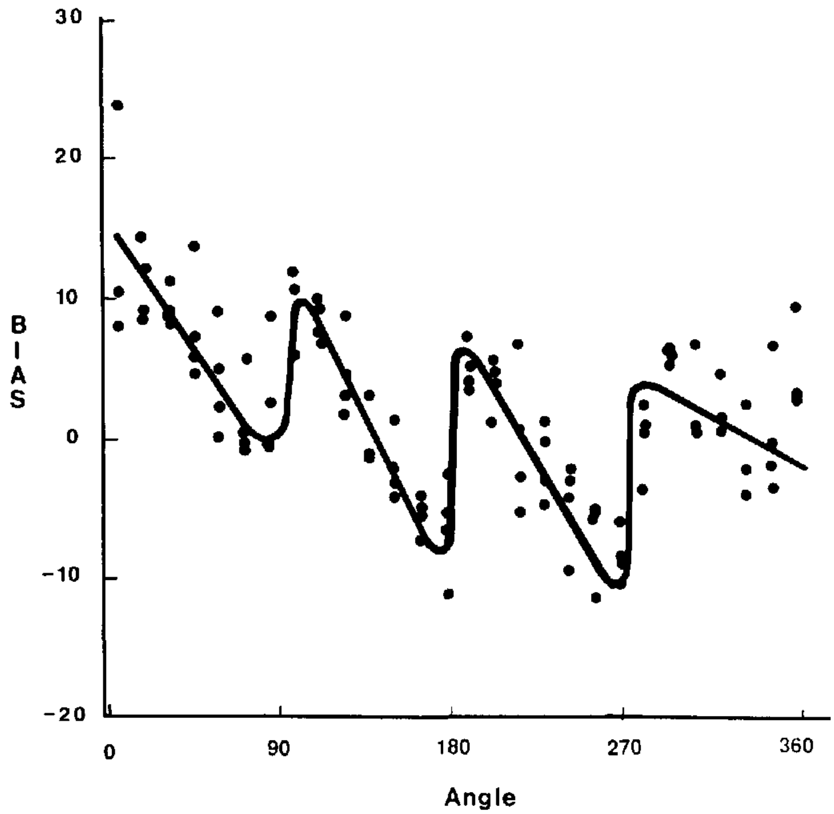

Figure 8. Mean angular bias in responses as a function of stimulus location in Experiment 3. (The curve is the modeled estimate of angular bias as a function of angular location.)

locations was .035. Therefore, radius and angle again appear to be independent features of coding.

Bias in angular reports. The plot of the mean angular bias versus the actual angular location in Figure 8 shows a pattern of quadrant organization similar to that found in Experiments 1 and 2. Yet there is a difference in the pattern of angular bias near the quadrant boundaries between the present results and those in Experiments 1 and 2, notably a curvature or "hook" in the relation between mean angular bias and actual angular location in each quadrant. We hypothesize that the hook is a consequence of the fact that the quadrant boundaries, imposed by the subjects, are imprecise. For stimulus points near the quadrant boundary, some subjects will classify a stimulus dot as being in one quadrant, whereas others will classify it as being in the adjacent quadrant. In Experiment 2, the angular locations nearest the boundary were 8 degrees away, and only 2.7% of the reports misclassified stimuli into the adjacent quadrant. In contrast, in Experiment 3 where the actual angular locations nearest the boundary were only 6 degrees away from it, 10.6% of the reports of angular locations nearest the boundary were misclassified.

Subjects' responses will depend on the quadrant in which the subject classifies the stimulus. That is, reports by subjects who misclassify the stimulus will be weighted with the prototype for the quadrant in which they classify the stimulus. Thus, the reports for locations nearest the boundary reflect some individuals who classify the stimulus into the correct category and whose responses correspond to the linear relation between actual location and reports for that category. Some of the reports (an average of 10.6%) are misclassified, and reflect subjects whose responses correspond to the linear relation for the *adjacent* category. Our regression model for uncertain boundaries estimates the regression coefficients in each category under the assumption that the mean report at each location is generated by a combination of some reports that are correctly classified (and generated by the correct quadrant's linear regression) and some that are incorrectly classified as being in the adjacent quadrant (and generated by that quadrant's linear regression). The proportions of each are determined by the proportion of misclassifications. Note that the model for uncertain boundaries produces results identical to simple regression analyses within quadrants when applied to the data from Experiments 1 and 2 where no points had angular locations near quadrant boundaries.

The curve in Figure 8 is the estimate derived from our model for uncertain boundaries (see the Appendix). Using this model the estimated regression slopes are $b = -.22, -.28, -.24$, and $-.09$ for Quadrants I, II, III, and IV, respectively. The estimated angular locations of zero bias are $\theta = 75$ degrees, 138 degrees, 214 degrees, and 330 degrees, respectively (again only the neutral point in the upper right quadrant differs from the middlemost value). Note that the pattern of these estimated values from the model closely parallels that of the empirical results in Experiments 1 and 2.

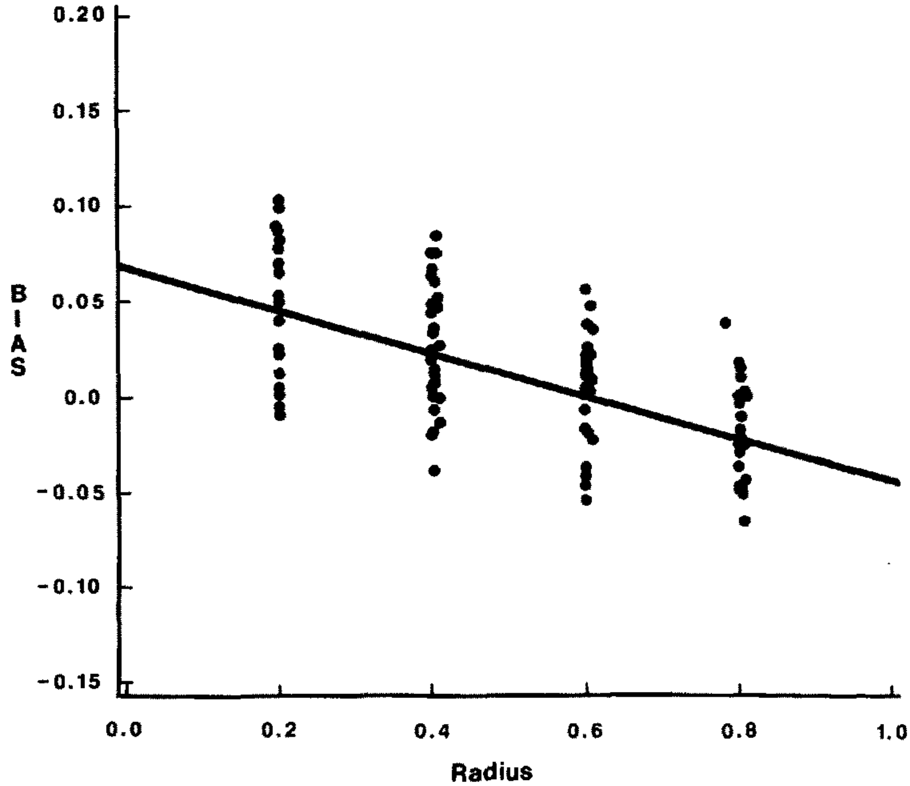

Figure 9. Mean radial bias in responses as a function of stimulus location for Experiment 3. (The solid line is the regression of radial error on radial location.)

Bias in radial reports. The general pattern of radial bias as a function of actual radius is similar to that found in previous experiments. The standard deviation of reports of radial location was again strongly related to actual radial location, $r = -.53$, $p < .05$. Figure 9 shows that radial bias is positive (away from the center of the circle) for locations near the center, and has a smaller negative (inward) magnitude for actual locations near the circumference. Here, however, the radial location corresponding to zero bias is nearer the center than in the previous experiments. Regressing radial error on actual radius yields a slope estimate of $b = -.113$, $p < .01$. Using this regression line to estimate the radial location corresponding to zero bias yields the radial location $r = .61$, and the 95% confidence interval for the location with zero bias (computed using Feiller's theorem) is .59 to .63. Clearly, the radial prototype is not near the circumference line as in Experiment 1. Indeed, the prototype was different both from .7, the location of the radius in a circle that contains half the area of the full circle, and from the average radial location of the stimuli, which was .5. It seems that the value of the radial prototype may depend on both middlemost value (i.e., which divides the area of the circle in half) and on the average presented value; as mentioned previously, the details of how a prototype value arises are not explored in the present article.

Advantages of the use of angular and radial prototypes. The typical standard deviations of angular and radial reports were essentially the same as those obtained in Experiments 1 and 2, as were the slope of angular and radial bias regressed on actual angular and radial locations. Hence, as in Experiments 1 and 2, the use of angular and radial prototypes produced more accurate reports than those based on fine-grain recollections alone.

Experiment 4

According to the model, the relative weights of the particular value and the prototype depend on the preciseness of information at each level. The weight of the prototype should increase when the dot coordinates, which are particular to a trial, are less exactly remembered. The fourth experiment tests this aspect of our model by introducing a visual interference task on each trial after presentation of a dot and before the subjects' response. As memory for particular locations becomes more uncertain because of the distractor task, the relative weight of the prototype should increase. We used a within-subjects design, comparing performance on the dots task alone, as structured in the first three experiments (standard trials) versus performance on the dual task of remembering dot location and performing a distractor task (interference trials).

Method

Subjects. The 31 subjects in this study were selected in the same way as those in Experiments 1, 2, and 3.

Materials. The stimuli for the standard trials consisted of eighty

8.5 × 11-in. (21.59 × 27.94 cm) sheets of white paper, each with a circle and a dot printed on it. The to-be-remembered locations were in the same positions as the basic set of 80 sheets described for Experiment 2. Subject comments during the previous experiment persuaded us that the 40 filler locations were unnecessary to achieve the illusion of even coverage of the circle.

As in earlier experiments, five random orders were used. The stimulus display apparatus was placed on a platform on the table in front of the subject, putting it at approximately the same height as the CRT in Experiment 2. The response surface slanted down from the display apparatus toward the subject at the same angle as in Experiment 2. Responses were collected using the stylus and digitizing pad as in Experiment 2.

The stimuli for the distractor task consisted of the same eighty 8.5 × 11-in. sheets of white paper used in the standard trials, plus another 80 sheets, on each of which was printed a 15-cm stimulus square, divided by lines into a 4 × 4 grid. Of the 16 square units within each stimulus grid, 8 were randomly filled in with black, creating a pattern of black and white units. Following each stimulus grid sheet was another sheet displaying a response grid containing only the 4 × 4 grid, with no blacked-out units. One unit on each response grid was marked with a large *X*. The subject's task was to indicate whether the unit marked with the *X* on the response grid had been white or black on the preceding stimulus grid.

The stimulus and response sheets for each trial of the interference trials were arranged as follows: The circle with the dot in it came first, followed by the stimulus grid and the response grid. A sheet of light blue card-weight paper separated each sheet from the next one in the set. Clip-on document rings held the sets of stimuli to the display apparatus as before. Again, five copies of the original stimulus set were created, and the order of stimulus sheets within each set was randomized.

Procedure. In this experiment, stimuli were manually presented in the same way as in Experiments 1 and 3. For the standard trials, the procedure was the same as in Experiment 3. For the interference trials, the procedure was as follows. One second after the experimenter revealed the stimulus dot, a new computer tone was the signal to uncover the distractor task stimulus grid. Subjects studied the black and white grid for 2 s before a third tone signaled removal of the stimulus grid, together with the following blue separating sheet, thus revealing the response grid. There were 3 s in which to recall whether the indicated unit was white or black. A fourth tone signaled removal of the response grid. The subject then indicated the location of the stimulus dot seen before the intervening grid task. Finally, a triad of tones alerted the subject that the next trial was about to begin. Thus, a new trial occurred every 13 s.

Subjects were run first in the standard trial format and then, following a short break, in the interference trial format. Subjects were given five practice trials for each task. For the standard trials, instructions were the same as for all previous experiments. For the interference trials, subjects were told that we were "interested in finding out how accurately people can perform on two competing tasks."

Results

Outlying responses were culled in the same manner as in Experiments 1, 2, and 3. This resulted in deletion of 2.5% of the reports in the standard trials plus an additional 1.0%, which fell outside three standard deviations for angular or radial values. It resulted in deletion of 14.6% of the reports in the interference trials, plus an additional .9% which fell outside three standard deviations for angular or radial values. This high percentage of deletions in the interference trials is an effect of the distractor task. The 14.6% of errors falling outside a 45-degree range of the presented values are trials on which subjects forgot what they had seen. The purpose of this study is to evaluate whether bias slopes are steeper on interference trials than on standard trials (because of inexactness in the representation of particular values). To be sure that the pattern of results was not altered by excluding such a large proportion of responses, we calculated both angular and radial bias both with and without extreme values in comparing bias slopes in the two conditions. Although the observed values were noisier for the unculled data, the bias slopes for the interference trials are steeper for angle and radius in both culled and unculled data sets.

Independence of dimensions. Once again, the amount of angular bias bears no systematic relation to amount of radial bias. The average correlation between angular and radial bias was $r = -.01$ (not significant) with a variance component of .032 for the standard trials, and $r = .06$ (not significant) with a variance component of .029 for the interference trials. Therefore, radial and angular location again appear to be independent features of coding.

Bias in angular reports. For both standard and interference trials, we found the same general patterns of quadrant organization as in the previous experiments. The plots of mean angular bias versus actual angular location for standard and interference trials, respectively, are shown in Figures 10 and 11. Having developed the model that treats quadrant boundaries as uncertain (see Appendix) and applied it in Experiment 3, regression lines for both standard and combination trials were estimated using the model for both standard and interference trials.

Recall that we hypothesized that the slope of the regression line of mean angular error on angular location would be greater for the interference trials than for the standard trials. Table 1 shows the within-quadrant slopes of mean angular bias versus angular location. Note that the slopes for the standard trials are similar to those in earlier experiments. The slopes are significantly ($p < .01$) greater for the interference trials than for standard trials in Quadrants I, II, and IV; the difference is in the predicted direction and approaches statistical significance in the other quadrant (III). This confirms the hypothesis that the prototype is given significantly greater weight in constructing responses when the distractor task increases the uncertainty of memory for the particular. The estimated angular locations with zero bias are $\theta = 55.3$ degrees, 169.4 degrees, 230.70 degrees, and 307.1 degrees for Quadrants I, II, III, and IV, respectively, in the standard trials and $\theta = 49.4$ degrees, 137.7 degrees,

Table 1
Regression Slopes of Angular Bias on Angular Location for Standard and Interference Trials (Experiment 4)

Quadrant	Standard	Interference	*t* s[a]
I	−.098	−.433	5.489*
II	−.164	−.344	2.885*
III	−.254	−.344	1.312
IV	−.215	−.373	2.601*

[a] The *t* statistic for testing the significance of the difference between regression slopes in standard and combination trials.
* $p < .01$.

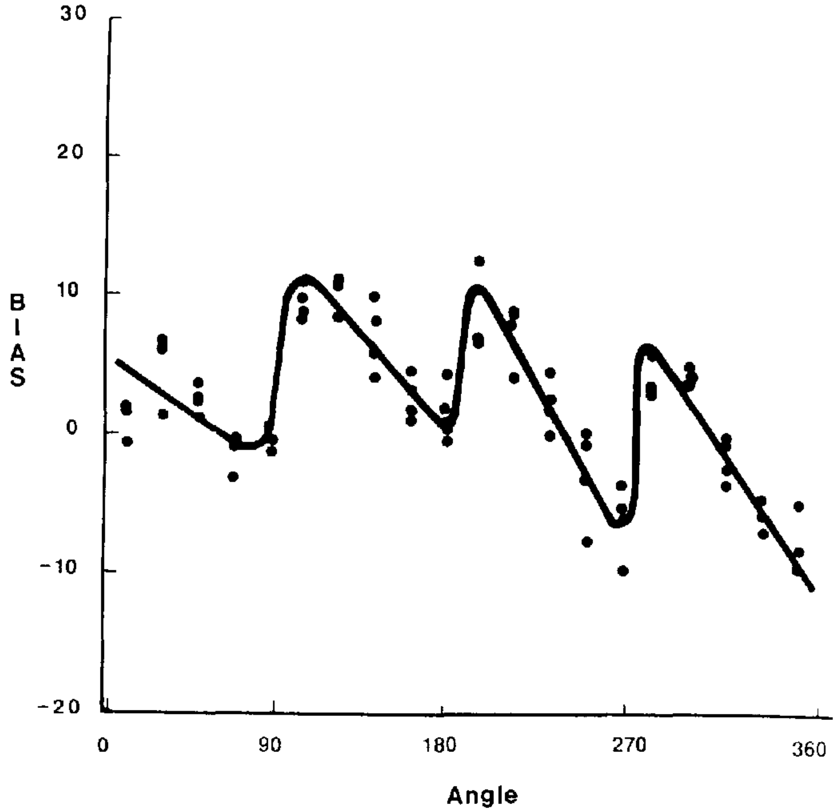

Figure 10. Mean angular bias in responses as a function of stimulus location for standard trials in Experiment 4. (The curve is the modeled estimate of angular bias as a function of angular location.)

226.5 degrees, and 311.7 degrees for Quadrants I, II, III, and IV, respectively, in the interference trials.

Bias in radial location. The plots of mean radial bias versus actual radial location for standard and interference trials, respectively, are shown in Figures 12 and 13. The general pattern of mean radial error as a function of actual radius for the standard trials is generally similar to that found in the previous studies, with positive (outward) radial bias for stimulus locations near the center, negative (inward) radial bias near the circumference, and a point of zero bias in between. The pattern for the interference trials is similar except that the slope for bias is generally steeper. Regressing mean radial location yields a regression slope of $b = -.11$ for standard trials and $b = -.36$ for interference trials. The difference between these two slope estimates is statistically significant, $t = 10.4$, $p < .001$. The estimated radial location zero bias is $r = .74$ (with a 95% confidence interval of .71 to .79) in the standard trials, and $r = .67$ (with a 95% confidence interval of .65 to .68) in the interference trials. Both of these locations are significantly greater than .60, the mean of the actual radial locations. The relation between standard deviation of radial reports and actual radius is again quite strong, $r = -.95$ ($p < .01$) in the standard trials and $r = -.90$ ($p < .01$) in the interference trials.

Discussion

In this article we have presented a model of category effects on reports of particulars. When people must report a particular stimulus value and their memory is imprecise, they combine remembered values with category information. Category information is used in estimation in two ways. First, remembered values are weighted with category prototypes. Second, estimates are constrained to fall within category boundaries. According to the model, the amount of bias in reporting can be predicted from the degree of inexactness in the representation of particular values and of category values (prototypes and boundaries). The use of an estimation process in which information is combined across levels, we have argued, can improve the accuracy of reporting.

We evaluated the model here in a case where subjects report the location of an item in a bounded space (a dot in a circle). We argued that subjects encode a dot as having both a fine-grain location (in polar coordinates) and a coarse-grain location (a quadrant). The evidence of quadrant coding consisted of a pattern of angular bias away from the horizontal and vertical axes and of radial bias away from the circle's center (and to a lesser extent away from the circumference). The evidence of use of prototypes in estimates of location consisted of a pattern of bias toward a neutral or prototypic value within the category for both radius and angle. When the precision of memory for particular values was decreased by presenting a distractor task, the prototype was weighted more heavily, leading to steeper slopes for both angular and radial bias, as predicted by the model. In all cases, the use of a prototype in reporting increased the overall accuracy of estimation.

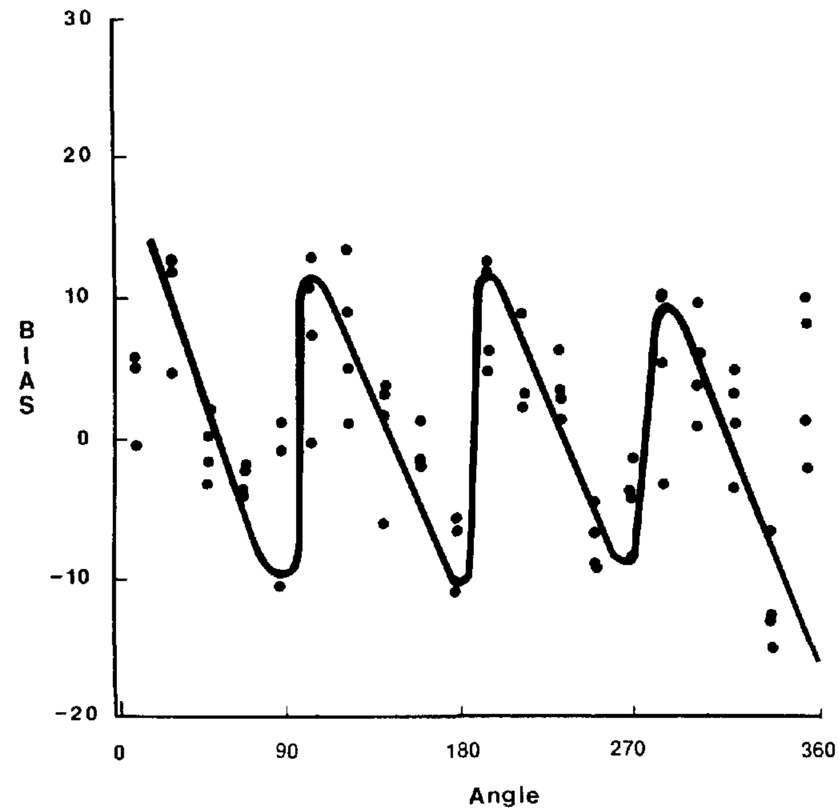

Figure 11. Mean angular bias in responses as a function of stimulus location for interference trials in Experiment 4. (The curve is the modeled estimate of angular bias as a function of angular location.)

When Is Category Information Used in Estimation?

In this article we have explored a function of categories not previously examined in the literature (i.e., to improve estimates of particular stimulus values). In our studies, weighting with a prototype does indeed improve estimation (i.e., on average, reports are more accurate when the prototype is used). However, we do not yet know the extent to which category information would be used in estimation if it did not improve accuracy or even decreased accuracy. That is, to form representations at two levels and combine information across levels in estimation may be a process used widely across contexts because it often improves accuracy of reports. Alternatively, the process may be context sensitive, used only when subjects believe it will improve estimation.

There is some evidence that context can affect whether or not people construct ad hoc spatial categories and what categories they construct. B. Tversky and Schiano (Schiano & B. Tversky, 1989; B. Tversky & Schiano, 1989) found context effects on bias in a spatial location task. They presented a spatial frame consisting of a horizontal line and a vertical line forming a right angle with a stimulus line extending outward from the corner in different orientations on particular trials. Subjects then were presented the frame and drew a line in the orientation they had been shown. When the frame was described as a map, no bias was found. When the frame was described as a graph, there was a pattern of bias similar to ours (misplacement toward 45 degrees). When no information about the frame was provided, there was bias away from both the diagonal and the horizontal and vertical axes toward points at roughly 22 degrees and 67 degrees.

We (but not the authors) tentatively interpret these context effects as revealing when and how people may categorize a space. When the frame is described as a map, the space is conceptualized at only one level of detail because, in a map, avoiding bias at each location is more important than improving overall accuracy at the cost of bias at some locations. In the absence of instruction, subjects divide the frame into two categories, with a boundary in the middle (at the diagonal) and a prototype near the center of each of these categories (at roughly 22 degrees and 67 degrees). When the frame is described as a graph, subjects treat it as a subdivision of a larger space consisting of four quadrants (categories); the frame is treated as one of the categories, with a prototype near the center (at the diagonal).

Representation of Category Structure

The assumptions of our model and the predictions it makes are relevant to the understanding of category structure. In fact, the claims of the model concern two issues that have received considerable attention in the existing literature.

Graded effects. In the recent literature, there has been considerable interest in what is called the *graded structure* of catego-

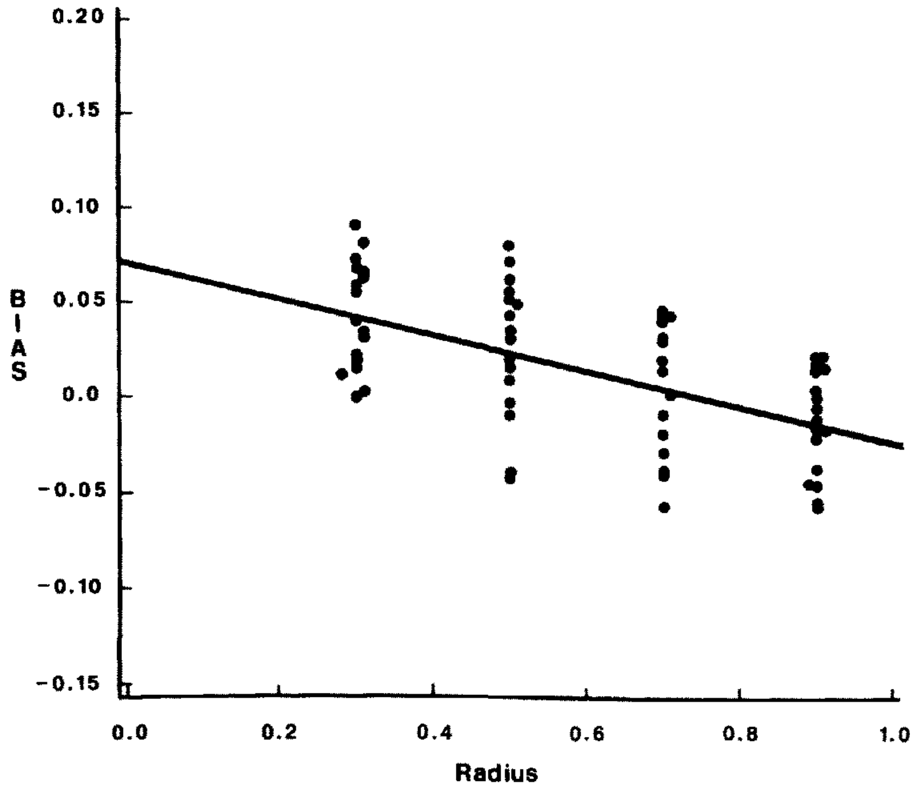

Figure 12. Mean radial bias in responses as a function of stimulus location for standard trials of Experiment 4. (The solid line is the regression of radial bias on radial location.)

ries (cf. Smith & Medin, 1981). The classic view is that categories simply specify necessary and sufficient conditions for membership. Findings that indicate that stimuli differ in how good they are as category members have been treated as evidence against this classic view. Such findings include the fact that, for different stimuli, there are differences in the amount of time it takes to judge whether or not that stimulus is a category member, and that people judge different stimuli to be better or worse exemplars of a category. Let us consider how such findings can be explained in the context of the model we have proposed.

According to the present model, stimuli are represented at two levels of detail. These can be thought of as values in two isomorphic spaces; in one, a stimulus is represented as a point corresponding to its particular value, and in the other, as a region corresponding to its category. Each of these two isomorphic spaces has an image in the other. To judge if a particular stimulus is a category member, a person must locate its value relative to the category boundary. Thus, even if the region (category space) is not itself graded, variation in the position of an item in the category (nearness to the boundary) can lead to graded effects of the sorts observed. Furthermore, the boundaries of a region (category) may be more or less exact, and this will affect the probability of assigning items to that category. Rather than reflecting a gradation of the category space, such imprecision may reflect uncertainty about boundary location within that category space.

Status of prototypes and exemplars. In the existing literature, there has been considerable discussion of whether category and exemplar (particular stimulus) information are represented separately in memory. In a landmark article, Posner and Keele (1968) argued that people abstract prototypes when they form categories from the presentation of sets of exemplars. The sort of evidence that has been offered for prototypes concerns the differential treatment of different stimuli, that is, the graded effects noted previously here. In particular, after learning a category from a sample of stimuli, subjects are likely to recognize (believe they have seen) stimuli near the mean of presented values (even if they have not seen them), and may categorize stimuli near the mean better than others. Later investigators (cf. Brooks, 1978; Medin & Schaffer, 1978) proposed an alternative model according to which people store category exemplars and do not generate category-level information. There is evidence that people do retain particular stimuli, which some investigators regard as evidence against prototype models. Although showing that a model that posits that prototypes (or boundaries) alone are represented is not adequate, such findings do not distinguish between models that posit only exemplars and those that, like our model, posit both exemplars and category-level information.

Exemplar models explain the ability to assign new items to categories as the result of a sampling process by which those items are compared with exemplars of the category stored in memory. For example, if a new item has a value falling outside the range of values of the set of instances sampled, it will not be considered to be a category member. Thus, the sampling pro-

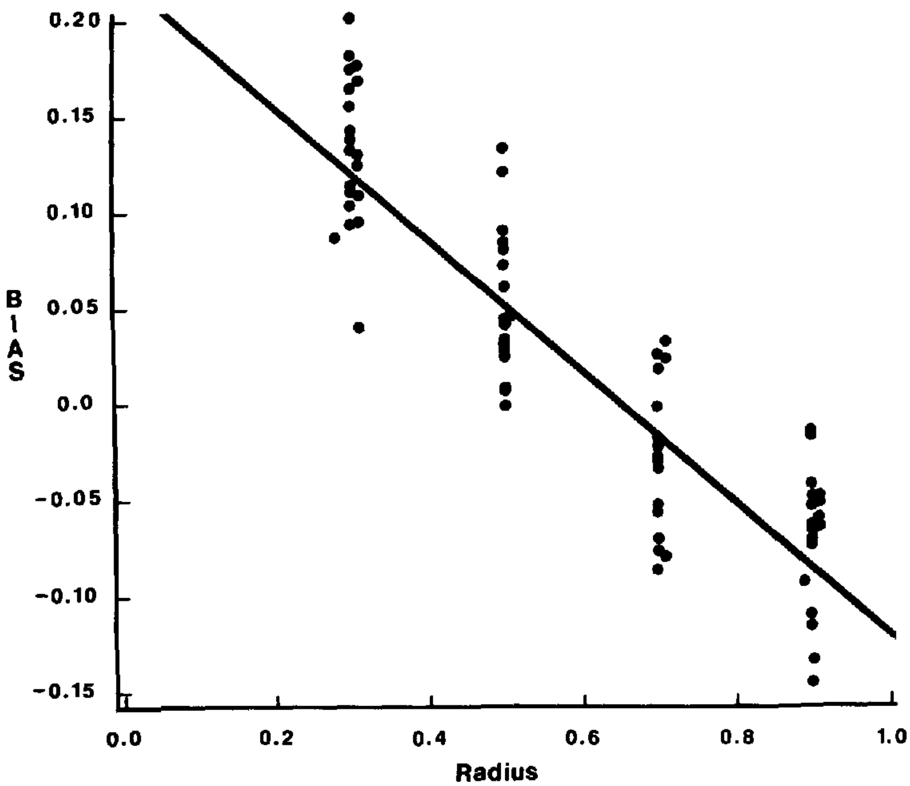

Figure 13. Mean radial bias in responses as a function of stimulus location for interference trials of Experiment 4. (The solid line is the regression of radial bias in radial location.)

cess gives rise to boundary information without explicit representation of boundaries. Estes (1986) argued that the critical difference between prototype models and exemplar models lies in when and how category information is made available: early (i.e., explicitly represented in memory) or late (i.e., arrived at implicitly by means of sampling of exemplars). Existing experiments do not permit differentiation of early calculation from late calculation models. Our data show that people's reports of particular experiences are based on a weighted combination of a prototype and a particular value; it seems most straightforward to explain such a result by positing that category prototypes are explicitly represented in memory before the process of constructing estimates.

Representation of Physical Scales

Bias in spatial reports frequently has been taken to indicate that spatial representation itself is biased. However, our model shows how biased reports can be generated from unbiased multilevel representations. Next, we consider the model's implications for certain arguments in the literature. First, the model predicts asymmetry in similarity judgments depending on the direction in which two items are compared. Second, the model predicts two well-known phenomena in psychophysics: contraction bias and the bias captured by Weber's law.

Asymmetry of similarity (distance) judgments.[4] There are sometimes asymmetries in judgments of similarity or spatial distance. For example, people may be asked the distance from *A* to *B* or from *B* to *A*, or to draw the location of one item on a map that correctly displays the location of the other item. The fact that people's reports may depend on the direction of comparison has led investigators to argue that representation of information in memory is not metric. A. Tversky (1977) suggested that similarity (distance) judgments should instead be thought of as arising from a comparison of features. He showed that a feature comparison model can be used to judge similarity (distance) even when the objects compared lie on a continuum. Such a model is not constrained to yield symmetric similarities, so it can accommodate asymmetries in similarity judgments. Nosofsky (1991) recently argued that a model such as Tversky's can always be derived from one that posits an underlying representation of similarity, which is symmetric, plus a bias associated with each of the items to be compared. Although Nosofsky demonstrated that stimulus bias can generate asymmetry in similarity judgments, he did not provide an explanatory framework for such biases. Our model does just that.

According to our model, asymmetry in similarity (distance) judgments arise because estimates of stimulus values are adjusted to reflect category information. Consider a comparison of two items in a category where one item is near the center and the other is near a boundary. One item is fixed, and subjects

[4] Nora Newcombe, a collaborator in studies of spatial development, has been involved in our discussions of this issue.

must either locate the other item or estimate its distance from the fixed item. First, consider the case where the center item is fixed at its true location. Because the location of the item near the boundary is "shrunk" toward the center, distance will be underestimated. Next, consider the case where the item near the boundary is fixed at its true location. Because the location of the central object is relatively unaffected by boundary or prototype effects, there is not a corresponding underestimation of distance. Hence, our model predicts asymmetries in judgments of distance (similarity) depending on which object is fixed.

Contraction bias. It is well known in the psychophysical literature that discrepancies between people's reports of stimulus values and presented values reflect what has been called the *central tendency of judgment,* first described many years ago by Hollingworth (1910). This tendency of reports to "gravitate toward a mean magnitude," which Poulton (1979) called a contraction bias, is found across a wide range of dimensions. A well-known model that was proposed to explain this pervasive form of bias is Helson's (1948) adaptation-level model. His model, like ours, posits that people's reports of particular stimuli are based on the difference between the adaptation level and the stimulus value. Helson's model, which predates cognitive psychology, did not explain how the observed pattern of bias might arise. Furthermore, it is not possible to determine if the range of data Helson discussed can be explained by our category model, because his model is applied to stimuli whose values are already specified on a psychological rather than a physical scale.

Weber's law. Psychological judgment of physical differences between stimuli may vary at different points on a dimension, notably for dimensions involving physically increasing magnitudes. This relation is formalized in Weber's law, which holds that just noticeable differences in magnitude are a constant proportion of that magnitude. Our studies deal with subjects' reports of stimulus values, not judgments of differences between stimuli. We find a decrease in accuracy of reports (i.e., an increase in the standard deviation) with physically increasing magnitudes. In the present study, where subjects reported radial location, accuracy of the representation decreased as distance from the circumference increased. In an earlier study, where subjects reported the elapsed days since a target event, accuracy of the representation decreased as actual elapsed time increased (Huttenlocher et al., 1990).

The increased variability of reports with increases in magnitude along a dimension in our studies would lead to changes in difference judgments (including just noticeable differences) at different points on a scale. Although our reporting tasks do not explicitly require discrimination between stimuli, discrimination is a natural extension of these tasks. In discrimination, a subject generates (internally) a report from an uncertain memory of stimulus A and compares it with a fixed standard B. The probability that A is reported as larger than B is a function of the true difference between A and B and the uncertainty (standard deviation) of recollection of stimulus A. If the uncertainty in memory (seen in the standard deviation of reports) increases with stimulus magnitude, the true difference required for discrimination with a fixed probability will be a function of magnitude. If the increase in standard deviation is linear, the resultant changes in discrimination will approximate the Weber fraction.

In some psychophysical studies, as in our studies, there is an increasing downward divergence of judgments from true values as magnitude along those dimensions increases. In our model, such divergence will arise if there is an increase in uncertainty as to particular stimulus values at larger magnitudes. The model posits two mechanisms to account for downward bias: shrinkage toward a prototypic central value and the use of *roundings* in the measurement of larger magnitudes.

In summary, the model we have presented can explain certain discrepancies between physical scales and people's reports of the sort described in the psychophysical literature. The model shows that systematic bias in reporting stimulus values can arise even if memory itself is unbiased. The general assumptions about memory made in the model are familiar: that memory is inexact and hierarchically organized, and that reports from memory are reconstructions. What is novel in the model is to posit estimation processes used to combine memory for particulars with schematic or category information in a way that predicts just the sorts of biases described in psychophysics. In cases where the model completely accounts for observed biases in reporting, it provides a cognitive explanation for the systematic distortions described by psychological scaling of physical dimensions.

Final Remarks

In the present article, we have presented a model of category effects on reports of particulars, and have applied the model to a case involving location in a simple bounded space. The model can be applied in a broader range of cases. First, although the case we examined had component dimensions that were independent, the model also applies in cases where the component dimensions are correlated. In such cases, information about one dimension contains information about the other dimension. Consequently, accuracy in estimating values on one dimension can be improved by incorporating information about values on the other dimensions. That is, use of a linear combination of information concerning values on correlated dimensions will improve estimation of values on the target dimension.

Second, in our case, categories were larger measurement units. Thus, availability of units at one level implied availability of units at all higher levels; for example, if a location is at 35 degrees (counterclockwise) from the right horizontal axis, it is necessarily in the upper right quadrant. The model also applies in cases where units are nested within higher level units that are not necessarily directly available by virtue of specifying lower units. This is true even in certain cases of spatial location. For example, consider the problem of finding your lost slides after a trip in which you gave a set of talks on some university campus. You know that you left them in the rear of a lecture hall near a projector, but you do not recall with certainty which lecture hall. In searching for the slides, you examine only rear locations in a set of lecture halls. Therefore, shrinkage toward a prototypic rear location specifies a multimodal distribution of locations in different rooms. A parallel situation occurs in temporal memory; it is possible to remember the time of day, but not the day of the week, when an event occurred. In this case, there will be a distribution of times around the true time of day, but because the day is unknown, the distribution of times will be multimodal around a particular time on different days.

A final point should be made about the model proposed here. That is, the model provides a way of discovering category structure from patterns of bias in people's reports. Given a set of reports from a stimulus domain, the pattern of bias in those reports will indicate regions corresponding to boundaries between categories, including information about the inexactness of those boundaries. In addition, the pattern of bias will indicate whether the category is structured around a prototype and where that prototype is located. Potentially then, the model we have proposed can be used to explore the ways in which people represent category structure.

References

Baird, J. C., Merrill, A., & Tannenbaum, J. (1979). Studies of the cognitive representation of spatial relations. II. A familiar environment. *Journal of Experimental Psychology: General, 108,* 92–98.

Bartlett, F. C. (1932). *Remembering: A study in experimental and social psychology.* Cambridge, England: Cambridge University Press.

Baum, D. R., & Jonides, J. (1979). Cognitive maps: Analysis of comparative judgments of distance. *Memory and Cognition, 7*(6), 462–468.

Belli, R. F. (1988). Color blend retrievals: Compromise memories or deliberate compromise responses? *Memory and Cognition, 16,* 314–326.

Bock, R. D., & Bargmann, R. E. (1966). Analysis of covariance structures. *Psychometrika, 31,* 507–534.

Brewer, W. F., & Nakamura, G. V. (1984). The nature and functions of schemas. In R. S. Wyer & T. K. Srull (Eds.), *Handbook of social cognition* (Vol. 1, pp. 119–160). Hillsdale, NJ: Erlbaum.

Brooks, L. (1978). Nonanalytic concept formation and memory for instances. In E. Rosch & B. Lloyd (Eds.), *Cognition and categorization* (pp. 169–211). Hillsdale, NJ: Erlbaum.

Cohen, A. C. (1950). Estimating the mean and variance of normal populations from singly and doubly truncated samples. *Annals of Mathematical Statistics, 21,* 557–569.

Deeley, J. J., & Lindley, D. V. (1981). Bayes empirical Bayes. *Journal of the American Statistical Association, 76,* 833–841.

Estes, W. (1986). Array models for category learning. *Cognitive Psychology, 18,* 500–549.

Hedges, L. V., & Olkin, I. (1985). *Statistical methods for meta-analysis.* San Diego, CA: Academic Press.

Helson, H. (1948). Adaptation-level as a basis for a quantitative theory of frames of reference. *Psychological Review, 55,* 297–313.

Hirtle, S. C., & Jonides, J. (1985). Evidence of hierarchies in cognitive maps. *Memory and Cognition, 13,* 208–217.

Hollingworth, H. L. (1910). The central tendency of judgment. *Journal of Philosophy, Psychology, and Scientific Method, 7,* 461–469.

Holyoak, K. J., & Mah, W. A. (1982). Cognitive reference points in judgments of symbolic magnitudes. *Cognitive Psychology, 14,* 328–352.

Huttenlocher, J., Hedges, L., & Bradburn, N. (1990). Reports of elapsed time: Bounding and rounding processes in estimation. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 16,* 196–213.

Huttenlocher, J., Hedges, L., & Prohaska, V. (1988). Hierarchical organization in ordered domains: Estimating the dates of events. *Psychological Review, 95,* 471–484.

James, W., & Stein, C. (1961). Estimation with quadratic loss. In L. M. LeCam, J. Neyman, & E. S. Scott (Eds.), *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability* (Vol. 1, pp. 361–379). Berkeley: University of California Press.

Johnson, N. L., & Kotz, S. (1970). *Distribution in statistics: Discrete distributions.* New York: Wiley.

Jöreskog, K. G. (1970). A general model for analysis of covariance structures. *Biometrika, 57,* 239–251.

Kosslyn, S., Pick, H. L., & Fariello, G. (1974). Cognitive maps in children and men. *Child Development, 45,* 707–716.

MacNamara, T. P. (1986). Mental representations of spatial relations. *Cognitive Psychology, 18,* 87–121.

MacNamara, T. P., Hardy, J. K., & Hirtle, S. C. (1989). Subjective hierarchies in spatial memory. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 15,* 211–227.

Maki, R. (1981). Categorization and distance effects with spatial linear orders. *Journal of Experimental Psychology: Human Learning and Memory, 7,* 15–32.

Medin, D. L., & Schaffer, M. M. (1978). Context theory of classification learning. *Psychological Review, 85,* 207–238.

Nelson, T. O., & Chaiklin, S. (1980). Immediate memory for spatial location. *Journal of Experimental Psychology: Human Learning and Memory, 6,* 529–545.

Newcombe, N., & Liben, L. S. (1982). Barrier effects in the cognitive maps of children and adults. *Journal of Experimental Child Psychology, 34,* 46–58.

Nosofsky, R. M. (1991). Stimulus bias, asymmetric similarity, and classification. *Cognitive Psychology, 23,* 94–140.

Posner, M. I., & Keele, S. W. (1968). On the genesis of abstract ideas. *Journal of Experimental Psychology, 77,* 353–363.

Poulton, E. C. (1979). Models for biases in judging sensory magnitude. *Psychological Bulletin, 86,* 777–803.

Sadalla, E. K., Burroughs, W. J., & Staplin, L. J. (1980). Reference points in spatial cognition. *Journal of Experimental Psychology: Human Learning and Memory, 6,* 516–528.

Schiano, D., & Tversky, B. (1989, November). *Structure and strategy in memory for line slope.* Paper presented at the 30th annual meeting of the Psychonomic Society, Atlanta, GA.

Shapiro, A., & Browne, M. W. (1987). Analysis of covariance structures under elliptical distributions. *Journal of the American Statistical Association, 82,* 1092–1097.

Smith, E. E., & Medin, D. L. (1981). *Categories and concepts.* Cambridge, MA: Harvard University Press.

Stevens, A., & Coupe, P. (1978). Distortions in judged spatial relations. *Cognitive Psychology, 10,* 422–437.

Thorndyke, P. W. (1981). Distance estimation from cognitive maps. *Cognitive Psychology, 13,* 526–550.

Tversky, A. (1977). Features of similarity. *Psychological Review, 84,* 327–352.

Tversky, B., & Schiano, D. (1989). Perceptual and conceptual factors in distortions in memory for graphs and maps. *Journal of Experimental Psychology: General, 118,* 387–398.

Wilton, R. N. (1979). Knowledge of spatial relations: The specification of the information used in inferences. *Quarterly Journal of Experimental Psychology, 31,* 133–146.

Appendix

A Mathematical Model of Combining Uncertain Information

Our model makes three important assumptions about representation. The first is that representations are *multilevel;* that is, items are represented simultaneously in terms of units of different sizes, and these representations are retained separately. The second is that these representations are *unbiased;* that is, the average or expected value of recollections is at the true value. The third is that representations are *inexact;* that is, a representation at any level can be characterized as a distribution with a particular standard deviation around the true value. The model holds that bias in reports from memory is a consequence of combining inexact information from different levels. Our spatial categories are two dimensional; consequently, they must be represented by two coordinates. In our experiments, we have found that the two dimensions (radius and angle) are sufficiently independent to be treated separately. Hence, the discussion presents a one-dimensional model that can be applied independently to each coordinate.

The model we have presented in this article is based on a set of statistical and probabilistic considerations. This can be most precisely and usefully stated in a mathematical form. This model is used to make precise the statements and predictions made in the body of the article. It is also used to estimate the slopes of the regression lines of reports of angular location on actual angular location in the model for uncertain boundaries. In this case, the parameters are estimated by the method of maximum likelihood. The sections that follow state the component parts of the mathematical form of the model and then combine them into the overall model. First, the model is presented treating boundaries as if they were precisely known. A model for the constraining (truncating) effects of boundaries is presented, and then a model for the effects of prototypes is presented. Next, the combined effects of prototypes and the truncating effects of boundaries are presented. Finally, the model is presented for the case involving uncertain boundaries.

Boundary Model

An effect of category boundaries arises because reports based on recollections from memory are constrained to lie within a category. Hence, the report distribution is a truncated form of the distribution of recollections from memory. If the distribution of the recollections is known, the bias at any actual location relative to a precise boundary can be computed analytically. The effects of imprecise boundaries are considered in a subsequent section.

Let the random variable M represent the recollection from memory of an object with true location μ. We assume that M is normally distributed about μ, with standard deviation σ_M corresponding to the uncertainty of the information encoded in memory. The notion that memory is unbiased is operationalized by the assumption that the mean of M is μ, the actual location. If reports R are generated from M by means of truncation at points a and b (that is, constraining R to lie between a and b), then the expected value of the report R is:

$$E[R] = \mu + \frac{[\phi(a_s) - \phi(b_s)]\sigma_M}{\Phi(b_s) - \Phi(a_s)}, \tag{A1}$$

where $a_s = (a - \mu)/\sigma_M$, and $b_s = (b - \mu)/\sigma_M$, and $\phi(x) = e^{-x^2/2}/\sqrt{2\pi}$ and $\Phi(x)$ is the standard normal cumulative distribution function. Hence, the bias of R is:

$$\text{Bias}[R] = E[R] - \mu = \frac{[\phi(b_s) - \phi(a_s)]\sigma_M}{\Phi(b_s) - \Phi(a_s)}. \tag{A2}$$

Note that if the actual location μ is more than two or three standard deviations from the boundary specified by a, then $\phi(a_s)$ will be very small. Similarly, if μ is more than two or three standard deviations from the boundary defined by b, $\phi(b_s)$ will be very small. If μ is more than two or three standard deviations from both a and b (e.g., in the middle of a category that is many standard deviations in width), then both $\phi(a_s)$ and $\phi(b_s)$ will be small, the right-hand side of Equation A1 will reduce to approximately μ, and R will be unbiased. Figure A1 shows the bias of R computed from Equation A2 as a function of the distance from the boundary in standard deviation σ_M units. It shows that the bias is substantial for locations near the boundary but is negligible for locations more than two standard deviations away from it.

Equation A1 uses the variance σ_M^2 of M to predict the expected value of R. Note that boundaries also have an effect on the variance of R. Consequently, when objects are located near boundaries, the variance of R will not provide an accurate estimate of the variance of M. However, this variance can be estimated from the variance of the reports R by means of the method of maximum likelihood using a truncation model (see, e.g., Cohen, 1950).

Prototype Model

Let the random variable M represent the recollection from memory of an object with true location μ. Let the random variable P represent the prototype location used by the subject to construct a response, and let ρ be the true mean of prototypes across subjects. This location is treated as random to incorporate the possibility that subjects are uncertain about the location of the prototype. We assume that the recollections from memory M have a distribution with standard deviation σ_M corresponding to their uncertainty. Because it is unbiased, the mean of the distribution of M is μ, the true location of the object. We also assume that the random variable P corresponding to the prototype location is distributed about an expected prototype location ρ, with standard deviation σ_P corresponding to the uncertainty of the prototype location. Note that it is not necessary to assume that the distributions of M and P have any particular form such as normality.

We posit that combining the recollection from memory M with information about category (prototype) P reflects the relative uncertainty of recollection from memory and prototype. Specifically, we suggest that:

$$R = \lambda M + (1 - \lambda)P, \tag{A3}$$

where λ is a weight that reflects the relative uncertainty of recollection and prototype. We posit that λ is an increasing function of $\theta = \sigma_P/\sigma_M$, and that λ tends to zero as σ_P/σ_M tends to zero. This corresponds to the assumption that the weight given to the recollection from memory increases as the relative uncertainty of prototypes to that of the recollection from memory increases. When the uncertainty of the memory is very large (compared with that of the prototype) so that memory provides essentially no information, we assume that subjects give essentially total weight to the prototype and essentially no weight to the memory (i.e., $\lambda = 0$).

Note that although the recollections from memory are unbiased, reports will generally be biased. This is because reports are produced by combining unbiased information from memory with generally biased information about category (prototype). The expected value of the report R is:

$$E[R] = E[\lambda M] + E[(1 - \lambda)P]$$
$$= \lambda E[M] + (1 - \lambda)E[P]$$

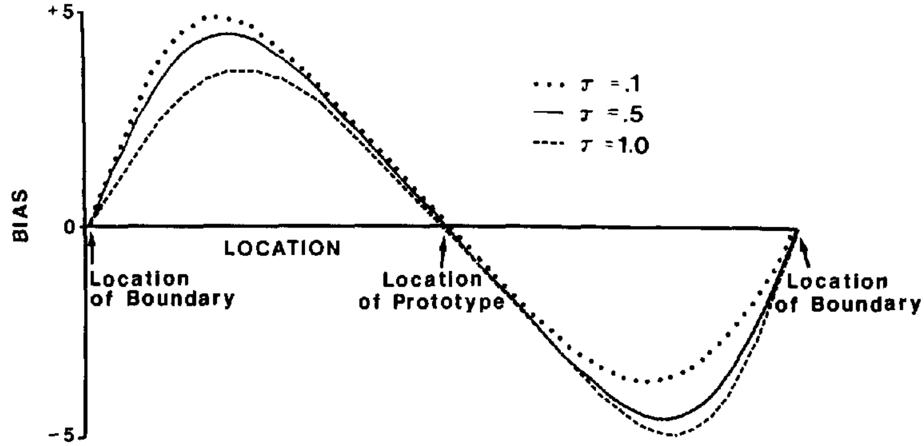

Figure A1. Bias in reports produced by truncation at perfectly certain boundaries as a function of stimulus location (in standard deviations of memory uncertainty from the boundary).

$$= \lambda\mu - (1 - \lambda)\rho. \quad \text{(A4)}$$

Thus, the bias of R (the expected value of the report minus the actual location) is:

$$\begin{aligned}\text{Bias}(R) &= E[R] - \mu \\ &= \mu(\lambda - 1) + \rho(1 - \lambda) \\ &= (\mu - \rho)(\lambda - 1). \quad \text{(A5)}\end{aligned}$$

Two general implications follow from the form of the bias given in Equation A5. First, the bias is exactly zero when the actual location of the object is at the prototype. Moreover, unless $\lambda = 0$, it is the only location within the category in which bias is zero; this provides a way to locate the prototype. Second, when the uncertainty in the location of the prototype is large relative to that of the recollection, the bias is small. This is because when σ_P/σ_M is small, λ is near 1. Conversely, when the uncertainty of location of the prototype is small relative to that of the recollection, the bias will be large.

Equation A5 also implies the form of the function relating bias to the actual location of the object. When the ratio σ_P/σ_M is constant for objects with different actual locations μ, then λ is constant across μ, and the bias is a linear function of μ with slope $\lambda - 1$ and intercept $\rho(1 - \lambda)$. When σ_M/σ_P is not constant across actual locations μ, then λ also changes with μ and the bias will not be a linear function of μ. The nature of the nonlinearity will depend on the way σ_P/σ_M (and hence λ) depend on μ, but qualitative predictions are generally possible given qualitative understanding of how σ_P/σ_M changes with μ and how λ changes with σ_P/σ_M. Quantitative predictions require knowledge of how λ changes with changes in σ_P/σ_M. Because the parameters of σ_P, σ_M, and θ are not observed directly, indirect means are needed to determine their values and the form of the dependence of λ on θ. The key to determining the values of λ and θ from data on subjects' reports is that the same prototype is used to construct reports for different objects in the same category. Hence, the reports must be correlated even if the recollections from memory of those objects are independent.

If each subject provides reports on several objects in a category, the magnitudes of the variances and covariances between observations are determined by the uncertainty of the memory for each object, by the λ coefficient for each object, and by the uncertainty of the prototype location. If the subjects each report the locations of k objects in the same category, there are $2k + 1$ parameters (k λ coefficients, k σ_M parameters, and σ_P) to be estimated, but k variances of reports and $k(k - 1)/2$ covariances between reports can be computed. Because the number of covariances grows much faster with k than does the number of parameters to be estimated, the information on variance and covariance can be used to estimate all of the parameters whenever $k > 3$. Estimation problems of this type are called analysis of covariance structures problems (see Bock & Bargmann, 1966), and although the particular problem that arises here has not previously been investigated, there is a considerable body of relevant statistical work that can be adapted to estimate σ_P and values of λ, σ_M, and θ for each location (see Jöreskog, 1970; Shapiro & Browne, 1987). Given estimates of θ and λ for each actual location, we can predict the bias at any point and empirically determine the relation between θ and λ.

A Model for the Combined Effects of Boundaries and Prototypes

The combined effects of boundaries and prototypes are derived from the results of the previous sections. We assume that the recollection from memory M combined with the prototype to produce a response is consistent with the prototype. That is, we require that the recollection M combined with prototype ρ_c for category c lie between the category boundaries a_c and b_c. This is equivalent to saying that M will have a truncated distribution determined by the limits of the category.

Combining Equations A1 and A4 yields the expected value of the response R (given that it is recalled as in category c)

$$E(R \mid C = c) = \lambda\mu + (1 - \lambda)\rho + \frac{[\phi(a_c^s) - \phi(b_c^s)]\sigma_M}{\Phi(b_c^s) - \Phi(a_c^s)}, \quad \text{(A6)}$$

where $a_c^s = (a_c - \mu)/\sigma_M$ and $b_c^s = (b_c - \mu)/\sigma_M$ and $a_c < b_c$ are the boundaries of category c. The bias of R (given that the category recalled is c) is

$$\begin{aligned}\text{Bias}(R \mid C = c) &= E(R \mid C = c) - \mu \\ &= \mu(\lambda - 1) + \rho(1 - \lambda) + \frac{[\phi(a_c^s) - \phi(b_c^s)]\sigma_M}{\Phi(b_c^s) - \Phi(a_c^s)} \\ &= (\mu - \rho)(\lambda - 1) + \frac{[\phi(a_c^s) - \phi(b_c^s)]\sigma_M}{\Phi(b_c^s) - \Phi(a_c^s)}. \quad \text{(A7)}\end{aligned}$$

Note that the right-hand side of Equations A6 and A7 differs from those of Equations A4 and A5 only by the last term involving a_c^s and b_c^s, the boundary locations in standardized units. If the actual stimulus location μ is more than a few standard deviations from both category boundaries, virtually all of the bias will arise from the effects of prototypes, and Equations A6 and A7 will reduce essentially to Equations A4 and A5.

Uncertain Boundaries

We posit that when boundaries are uncertain, subjects estimate or impute the location of the boundary and proceed as if the boundary location were known. Because the location of the boundary is uncertain, these boundary estimates will vary across items for the same subject and across subjects. Thus, when boundary location is uncertain, subjects will use a distribution of boundary values in constructing reports. Consequently, a distribution of effects on reports will be produced by an uncertain boundary. The expected value of the reports will be the average over the distribution of the potential boundary values of the effects that would be produced for each boundary value.

As indicated earlier, boundaries define categories. Thus, uncertain boundaries induce uncertainties in the assignment of a stimulus to a category. This is because some of the distribution of boundary values is consistent with classification of the stimulus in one category (e.g., just inside the boundary), whereas other stimulus values lead to classification of the object in an adjacent category (e.g., just outside the boundary). That is, some individuals may impute a boundary value that leads to classification of the stimulus in Category 1, whereas others may not.

Consider two adjacent Categories 1 and 2. Stimulus locations $\mu \leq B$ are in Category 1 ($C = 1$) and stimulus locations $\mu > B$ are in Category 2 ($C = 2$). We operationalize the idea that B is uncertain by treating B as a normally distributed random variable with mean β and standard deviation τ. Assume that the categories are sufficiently large (or τ is sufficiently small) that the probability is zero of classification into any category other than 1 or 2. Then the probability $P\{C = i|\mu\}$ that a stimulus with location μ is classified in category i is

$$P\{C = 1|\mu\} = \Phi\left(\frac{\mu - \beta}{\tau}\right),$$

$$P\{C = 2|\mu\} = 1 - \Phi\left(\frac{\mu - \beta}{\tau}\right). \quad \text{(A8)}$$

The expected value of the response R for stimulus at location μ is obtained by averaging the expected value of the report over the distribution of boundary values for each category choice and then combining the expectations according to the probability of classification into each category. This yields:

$$E(R) = E(R|C = 1)P\{C = 1\} + E(R|C = 2)P\{C = 2\}. \quad \text{(A9)}$$

Here the expected values of the report given classification into each of the categories are

$$E(R|C = 1) = \int_{-\infty}^{\infty} \left\{\lambda_1\mu + (1 - \lambda_1)\rho_1 + \frac{[\phi(a_1^s) - \phi(B^s)]\sigma_M}{\Phi(B^s) - \Phi(a_1^s)}\right\} \frac{\phi\left[\frac{(B - \beta)}{\tau}\right]}{\tau} dB,$$

$$E(R|C = 2) = \int_{-\infty}^{\infty} \left\{\lambda_2\mu + (1 - \lambda_2)\rho_2 + \frac{[\phi(B^s) - \phi(b_2^s)]\sigma_M}{\Phi(b_2^s) - \Phi(B^s)}\right\} \frac{\phi\left[\frac{(B - \beta)}{\tau}\right]}{\tau} dB, \quad \text{(A10)}$$

where $B^s = (B - \mu)/\sigma_M$, ρ_1 and λ_1 are the prototype location and the shrinkage coefficient in Category 1, and ρ_2 and λ_2 are the prototype location and the shrinkage coefficient in Category 2.

One consequence of Equation A9 is that when boundaries are uncertain, bias may be zero for locations at the expected value of the bound-

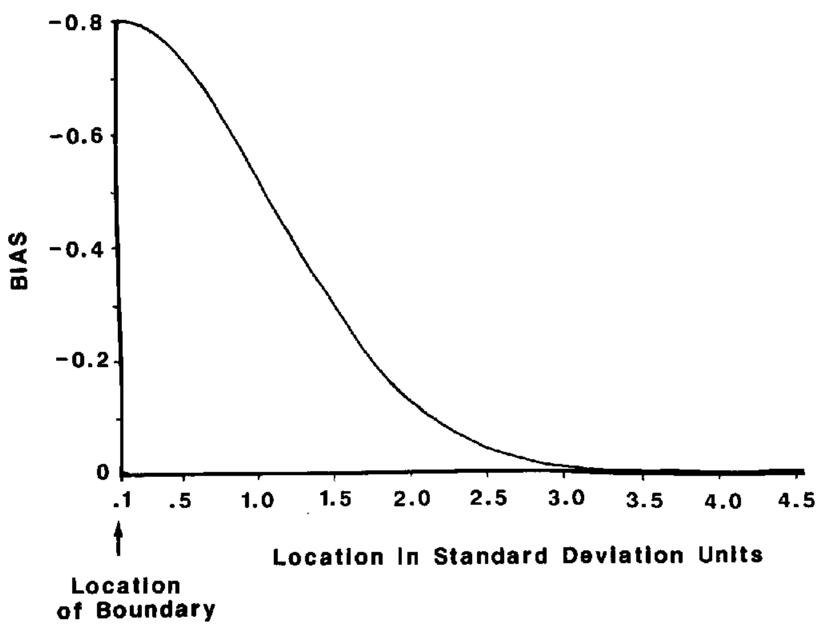

Figure A2. Bias in reports produced by the joint effects of truncation and prototypes for boundaries with normal distribution of uncertainty.

ary. This is because stimuli at such locations will be classified 50% of the time into each of the categories, and the bias introduced by both truncation and prototypes will be of opposite signs. If the prototypes are equidistant from the expected value of the boundary and the shrinkage coefficients λ_1 and λ_2 are the same, the bias for locations at the boundary will be exactly zero. If the prototypes are equally far from the expected values of the boundaries, then the bias will also be zero at a location near the prototype because the bias resulting from truncation will be small.

Figure A2 is a plot of the bias resulting from the joint effects of truncation and prototype as a function of actual stimulus location when both memory uncertainty and boundary uncertainty are normally distributed. Figure A2 shows the effect of changing the uncertainty τ (standard deviation) of the boundary location for a fixed memory uncertainty σ_M. It demonstrates that when there is small uncertainty about the boundary ($\tau = .1$), the bias is zero at the boundary and increases rapidly for locations between boundaries and the prototype, declining to zero again at the prototype location. However, when the boundary is more uncertain ($\tau > .5$ or 1.0), the maximum bias is not as large.

Bias Resulting From Truncation With Uncertain Boundaries

When boundaries are certain, bias resulting from truncation decreases rapidly with increases in distance from the boundary. If the standard deviation of memory for particular values is small, there should be a large area within the category for which bias is not introduced by truncation at the boundaries. When boundaries are uncertain, a person consults the range of potential boundaries consistent with the category assignment already made. Boundary values in this range are used in the truncation of memory for particular values. In this case, bias will be less near the boundary but will be found over a broader range (including values farther into the category) than is the case when boundaries are certain.

Consider a stimulus in a location with an underlying distribution of recalled values near the center of the distribution of potential boundaries. When the boundary is known precisely, the bias will be large. When the boundary is known imprecisely, boundary effects are an average of effects produced by a distribution of potential boundary values. Most of this distribution of potential boundaries will be farther from the actual stimulus location than the mean boundary and, hence, will produce smaller bias. The average of these bias values (the observed bias under imprecise boundaries) will thus be smaller than the bias near the mean, the bias observed with a precise boundary. Now consider an actual location far from the mean of the boundary distribution. If the boundary distribution has small variance (the boundary is precise), none of the potential boundary values would produce noticeable bias. However, if the boundary distribution has large variance (the boundary is imprecise), then some of the potential boundary values may be near enough to the actual location to produce substantial bias by means of truncation. This will lead to some overall bias because the observed bias is the average of effects reproduced by the distribution of boundary values.

The size of the bias resulting from truncation at imprecise boundaries can be computed exactly for inexact boundaries when the boundary uncertainty is known. Such computations confirm that the bias will be less near the boundary but will cover a broader range (including values farther into the category) than for exact boundaries. The biasing effects depend on the distance from the stimulus to the mean of the boundary distribution and on the uncertainty of both the particular values and of the boundary.

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